Average Speed

Speed, or more accurately average speed, is a measure of how fast an object is going. To calculate speed you must know the distance being travelled and the time taken to travel that distance. A simple rule can be used to calculate the average speed.

Average speed = distance travelled ÷ time taken

$$= \frac{\text{distance travelled}}{\text{time taken}}$$

Example 1. Find the average speed of an athlete who ran $7\frac{1}{2}$km in 20 minutes

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{7\frac{1}{2}\text{ km}}{20\text{ min}}$$

$$= \frac{7500\text{m}}{20\text{ min}}$$

$$= 375\text{ m/min}$$

Speed is usually written as:
metres/min,
metres/sec
or
km/hour.

The speed of an athlete could be given in units of km/hr as follows:

$7\frac{1}{2}$km in 20 min = $(7\frac{1}{2} \times 3)$km in (20×3) min

= $22\frac{1}{2}$km in 60 min

= $22\frac{1}{2}$km in 1 hour

= $22\frac{1}{2}$km/hr

ct is
e time
age

20

s

Example 2. Calculate the average speeds during Kalpeau's journey to the store in Exercise 2.1.

From the graph we can see that he was travelling at three different times.

a. Between 1:30pm and 1:40pm, Kalpeau travelled 500m.

Average speed $= \frac{500\text{m}}{10\text{min}}$

$= 50\text{m/min}$

b. Between 1:44pm and 1:50pm Kalpeau travelled 200m.

Average speed $= \frac{200\text{m}}{6\text{min}}$

$= 33\frac{1}{3}\ \text{m/min}$

c. Between 1:50pm and 2:00pm. Kalpeau travelled 300m.

Average speed $= \frac{300\text{m}}{8\text{min}}$

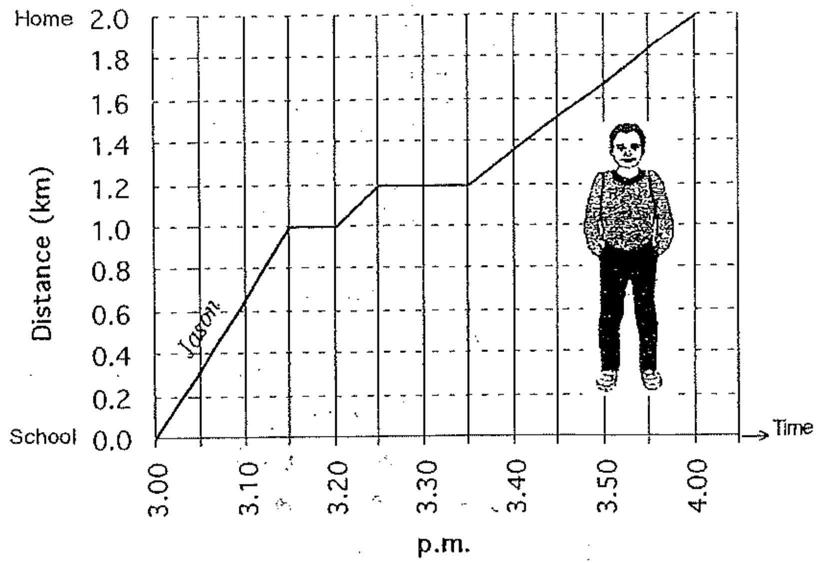
$= 37\frac{1}{2}\text{m/min}$

Note: Kalpeau's greatest speed of 50m/min occurred between 1:30pm and 1:40pm. Look at the graph. The line graph is the steepest between 1:30pm and 1:40pm.

Generally, the steeper the graph, the greater the average speed.

Exercise 2.2

Graph 1



1. How many stops did Jason make while walking home?
2. During what times was Jason not walking?
3. How far was Jason from school at 3:50p.m.?
4. How far was Jason from home at 3:50p.m.?
5. Calculate Jason's average speed between 3:00p.m. and 3:15p.m. in:
   a. km/hr
   b. m/min
6. a. What distance, *in metres*, did Jason travel between 3:20p.m. and 3:25p.m.?
   b. Calculate the average speed in m/min during this time.
7. a. What distance, in metres, did Jason travel between 3:35p.m. and 4:00p.m.?
   b. Calculate the average speed in m/min during this time.
8. Calculate the average speed for the total journey, that is, (total distance travelled) ÷ (total time taken).

Exercise 2.3 continued

Find the average speed of the following:

1. A horse runs 16km in 2 hours.
2. A truck covers 21km in a 3 hour delivery session.
3. An arrow travels 45m in $1\frac{1}{2}$ seconds.
4. A cyclist covers 35km in $1\frac{1}{2}$ hours, and rests for $\frac{1}{2}$ hour, and then covers an additional 25km in another hour.
5. A balloon is blown 32km in $2\frac{2}{3}$ hours.
6. An ant covers 84cm in 6 seconds.
7. An aeroplane flies 1500 metres in 10 seconds. (Find the speed in m/s and km/h.)
8. 200m covered in 5 seconds.
9. 180km covered in 90 minutes.
10. An athlete completes a 400m lap of a sports ground in 50 seconds.

Finding Distances

If Average Speed $= \dfrac{\text{Distance Travelled}}{\text{Time Taken}}$

then Distance Travelled $=$ Average Speed $\times$ Time Taken

Example If cyclists maintain a steady speed of 22km/h for 3 hours, how far will they travel?

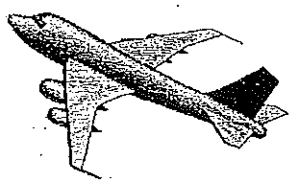
Distance = Speed × Time
= 22 km/h × 3hours
= 66km

Answer: The cyclists will travel 66 kilometres.

Finding Time

Time taken = $\dfrac{\text{Distance Travelled}}{\text{Average Speed}}$

Example A plane travelling at a speed of 600km/h is to travel 1350km. How long will this take?



$$\text{Time} = \frac{\text{Distance Travelled}}{\text{Average Speed}}$$

$$= \frac{1350\text{km}}{600\text{km/h}}$$

$$= 2\tfrac{1}{4}\ \text{h}$$

Answer: The plane will take 2h 15min to travel 1350km

Exercise 2.4

1. Find the distances travelled in the following:
   a. A person swims at a steady speed of 70m/min for 30 minutes.
   b. A beetle moving at a steady speed of 10cm per minute for 15min.
   c. A car travelling at 60km/h for $\frac{1}{2}$ hour.
   d. A truck moving at 90km/h for $7\frac{1}{2}$ hours.
   e. A ball rolling for 45 seconds at a steady speed of 28m/min. (Hint: time must be in minutes.)
   f. A motorbike travelling at 75km/h for $4\frac{1}{5}$ hours.
   g. A go-cart travels for 30 minutes at a speed of 20km/h, then 30 minutes at a speed of 26km/h.
   h. A jet moving at 400km/h has enough fuel to last for 6 hours.
2. Find the times taken for the following.
   a. Travel 350m at a speed of 70m/min.
   b. Run 1500m at a steady speed of 80km/h.
   c. A journey of 500km travelling at a speed of 7m/s.
   d. Completing a 1000km race at a steady 180km/h.
   e. Walking at 4km/h covering a distance of 26km.
   f. A truck leaves Alice Springs at 8:30a.m. and travels 320km at a steady speed of 80km/h, before stopping for a rest. At what time did the transport stop?
   g. In a long distance race Jack maintains a steady pace of 2m/s for 144km. How long would this take?
   h. A motorist averages a speed of 12km/h on her 18km trip to work. How long did the trip take?

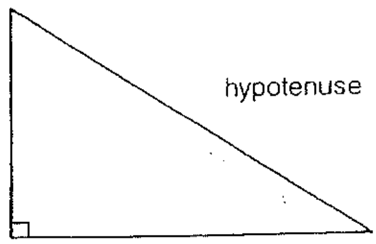
Topic Four: Pythagoras's Theorem

Pythagoras was a Greek mathematician who lived about 500 BC. At the age of fifty, he founded a religious society but little else is known about his life. The Pythagoreans were a secretive brotherhood whose symbol was the pentagram or 5–pointed star. Discipline was strict with members being silent for long periods and not allowed to eat meat or beans!

Pythagoreans investigated numbers and their patterns, invented the multiplication tables, discovered the maths in the musical scale and believed that all relationships in the physical world could be written as fractions.

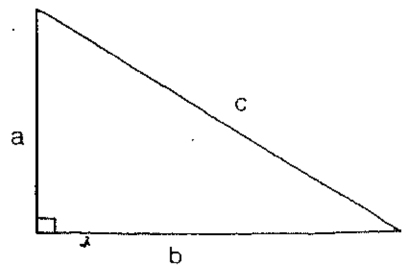
However, Pythagoras is best remembered for a theorem involving right-angled triangles.

For any right-angled triangle the longest side (opposite the right angle) is called the **hypotenuse**.



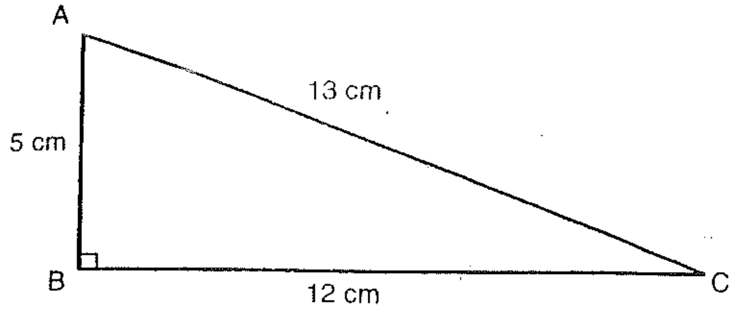
Pythagoras proved that for any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Pythagoras's Theorem



$$c^2 = a^2 + b^2$$

We can check Pythagoras's Theorem.



△ ABC is a right angled triangle.

AC = hypotenuse
= 13 cm

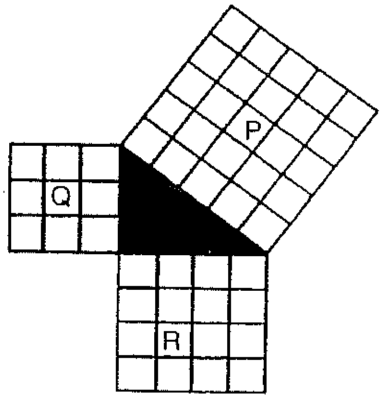
Hypotenuse squared = 13^2
= 169

Sum of the squares of the other two sides = $5^2 + 12^2$
= 25 + 144
= 169

They are the same!

therefore $(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

We can also see that the theorem is true by looking at the following diagram:



Counting squares ....

P (Hypotenuse squared) = 25 squares
Q = 9 squares
R = 16 squares

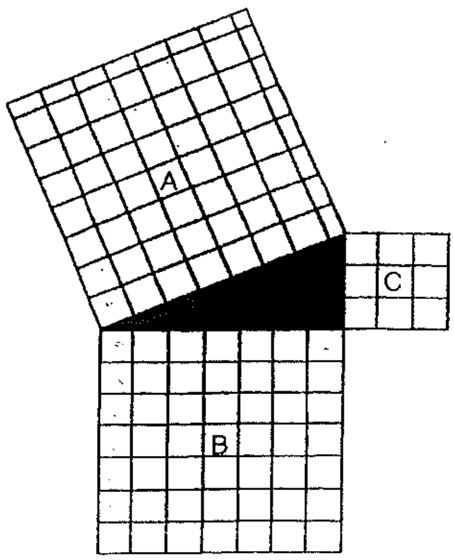
25 = 9 + 16

So P = Q + R

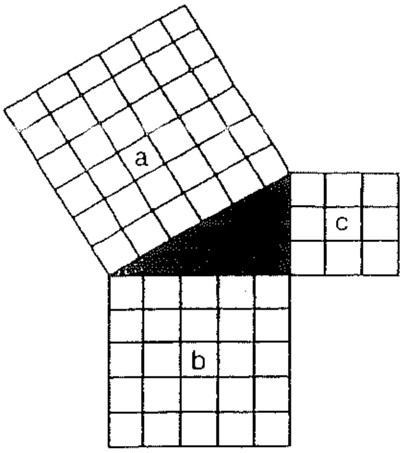
Exercise 4.1

1. Count the squares fill in the gaps

(a) B = ...
C = ...
A = 58
B + C = ... + ...
= ....



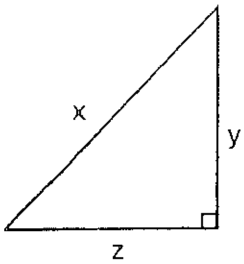
(b)



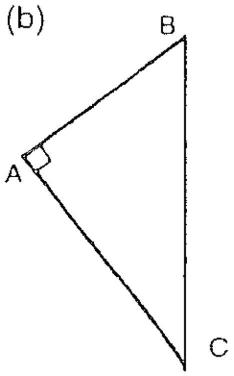
$a^2 = 34$
$b^2 =$ ....
$c^2 =$ ....
$b^2 + c^2 =$ ....

2. Name the hypotenuse in each of the following:

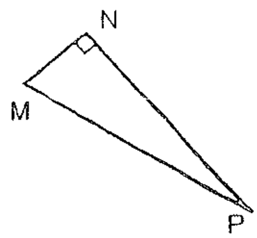
(a)



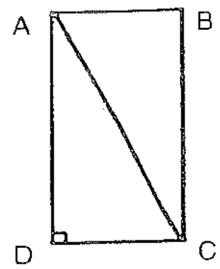
(b)



(c)

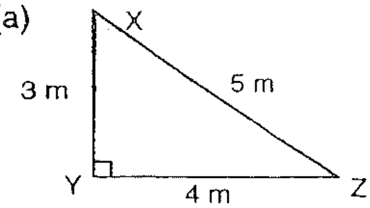


(d)

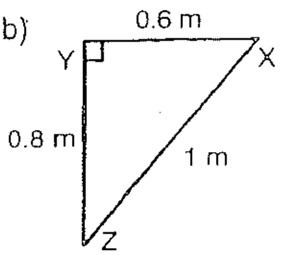


3. Copy and complete the table to check that Pythagoras's Theorem works for these triangles:

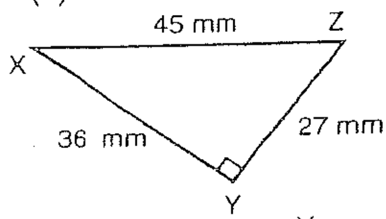
(a)



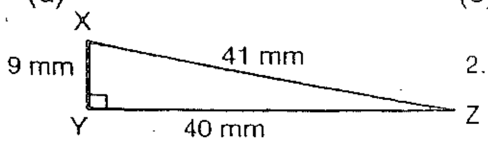
(b)



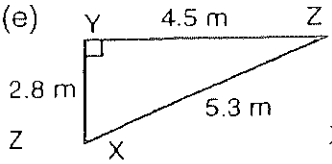
(c)



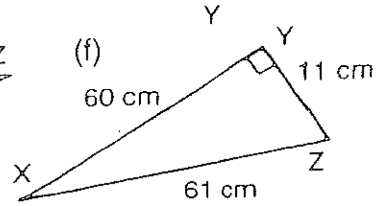
(d)



(e)

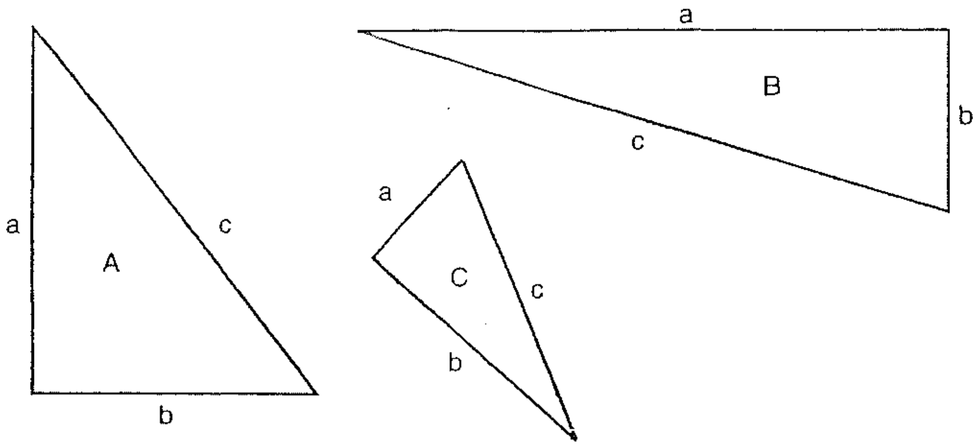


(f)



	$(\overline{XZ})^2$	$(\overline{XY})^2$	$(\overline{YZ})^2$	Pythagoras's Theorem $(\overline{XZ})^2 = (\overline{XY})^2 + (\overline{YZ})^2$
(a)	25	9	16	25 = 9 + 16
(b)				
(c)				
(d)				
(e)				
(f)				

4. Using your ruler, carefully measure the lengths of the sides of the following triangle and fill the table (after copying it into your workbook)

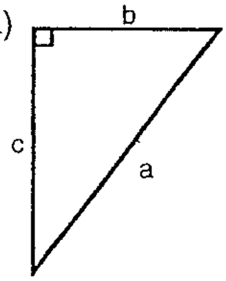


All measurements should be in millimetres.

	a	b	c	a^2	b^2	c^2	$a^2 + b^2$
A							
B							
C							

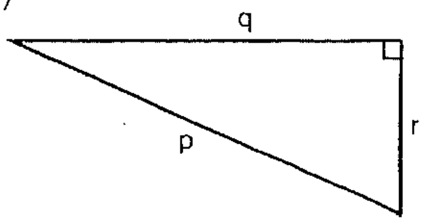
5. For each of the following triangles, choose the correct equation for Pythagoras's Theorem.

(a)



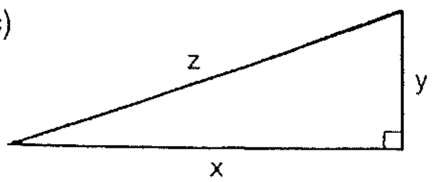
(i) $a^2 + b^2 = c^2$
(ii) $a^2 + c^2 = b^2$
(iii) $b^2 + c^2 = a^2$
(iv) $a^2 + b^2 + c^2 = 0$
(v) $b + c = a$

(b)



(i) $p^2 = q^2 + r^2$
(ii) $q^2 = p^2 + r^2$
(iii) $r^2 = p^2 + q^2$
(iv) $p^2 + q^2 = r^2$
(v) $p = q^2 + r^2$

(c)



(i) $z^2 + y^2 = x^2$
(ii) $x^2 + z^2 = y^2$
(iii) $x^2 - y^2 = z^2$
(iv) $z^2 = x^2 + y^2$
(v) $x^2 - z^2 = y^2$

6. Use Pythagoras's Theorem to check whether the following groups of numbers could be the sides of a right-angled triangle.

e.g. 6, 8, 10 — 10 must be the hypotenuse

$10^2 = 100$ — $36 + 64 = 100$

$6^2 = 36$

$8^2 = 64$ — ∴ Yes, a right- angled triangle.

(a) 1, 2, 3

(b) 10, 10, 15

(c) 11, 60, 61

(d) 18, 24, 30

(e) 7, 24, 25

(f) 16, 30, 34

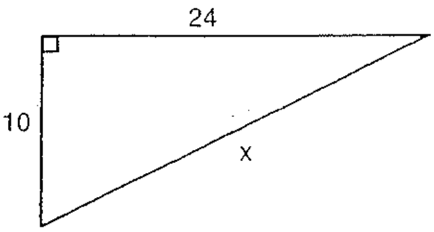
(g) 8, 17, 18

(h) 12, 19, 23

(i) 9, 11, 15

(j) 4, 5, 6

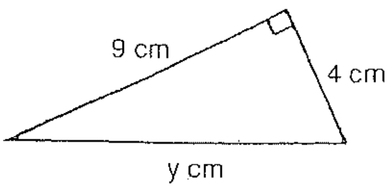
Finding the Hypotenuse



Steps

1. Write out Pythagoras's Theorem. — $c^2 = a^2 + b^2$
2. Substitute the values from the triangle into the theorem. — $x^2 = 10^2 + 24^2$
3. Calculate the squares. — $x^2 = 100 + 576$ $= 676$
4. Take the square roof of both sides to find x. — $\sqrt{x^2} = \sqrt{676}$, $x = 26$

Example: Find the value of the hypotenuse.



$c^2 = a^2 + b^2$

$y^2 = 4^2 + 9^2$

$= 16 + 81$

$= 97$

$y^2 = 97$

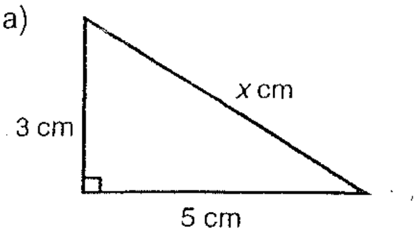
$y = 9.85$ cm

(correct to 2 decimal places)

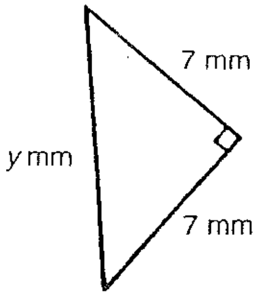
Exercises 4.2

1. Find the length of the hypotenuse, correct to two decimal places.

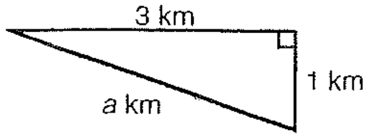
(a)



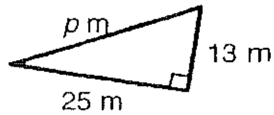
(b)



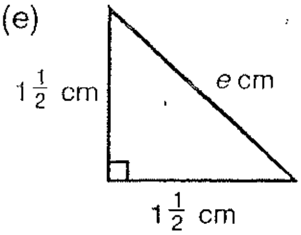
(c)



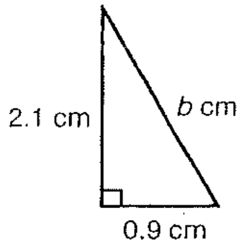
(d)



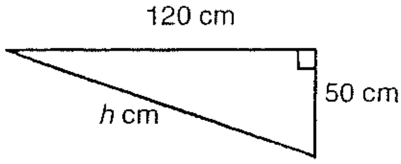
(e)



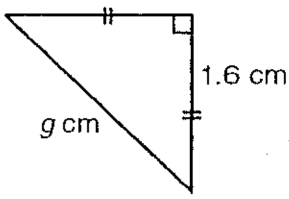
(f)



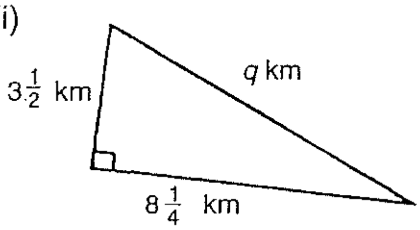
(g)



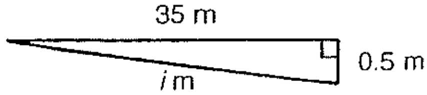
(h)



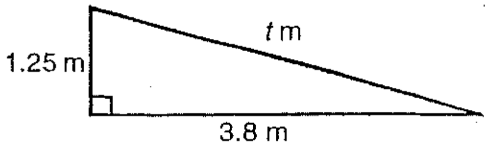
(i)



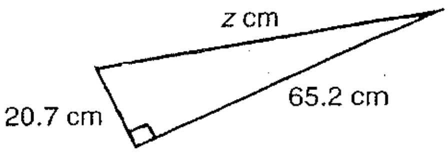
(j)



(k)



(l)

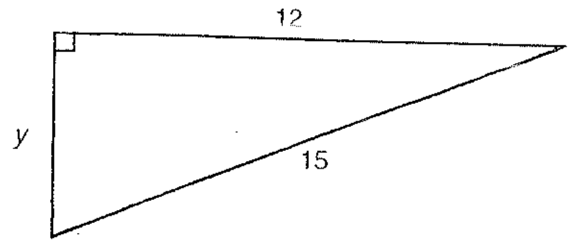


2. Find the length at the hypotenuse of a right-angled triangle with these smaller sides:

(a) 135 mm, 72 mm
(b) 144 mm, 42 mm
(c) 0.35 cm, 0.7 mm
(d) 0.38 cm, 0.1 cm
(e) 1 km, ¾ km
(f) 2 m, 1/5 m
(g) 5.6 cm, 22 mm
(h) 13.2 cm, 43 mm

* Careful with (g) and (h): units should be the same.

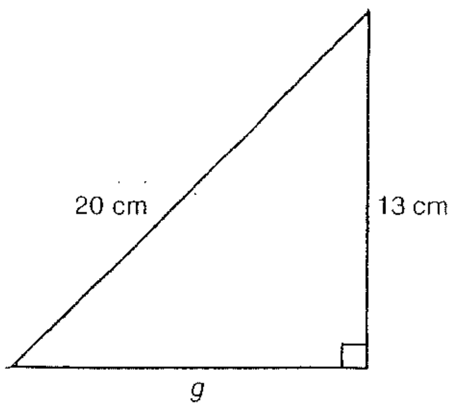
Finding a Short Side



Steps

1. Write out Pythagoras's Theorem. $c^2 = a^2 + b^2$
2. Substitute the values from the triangle into the theorem. $15^2 = y^2 + 12^2$
3. Calculate the squares. $225 = y^2 + 144$
4. Make y^2 the subject of the equation by subtracting 144 from both sides. $225 - 144 = y^2$
   $81 = y^2$
5. Take the square roof of both sides to find y. $\sqrt{81} = \sqrt{y^2}$
   $9 = y^2$

Example: Find the value of g, correct to three decimal places.

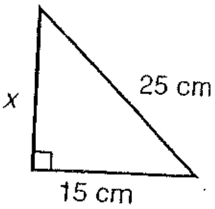


$$c^2 = a^2 + b^2$$
$$20^2 = g^2 + 13^2$$
$$400 = g^2 + 169$$
$$400 - 169 = g^2$$
$$\sqrt{231} = \sqrt{g^2}$$
$$15.199 \text{ cm} = g$$

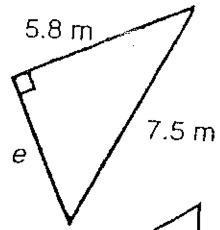
Exercise 4.3

1. Find the length of the unknown side (correct to 2d.p.).

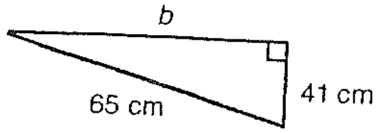
(a)



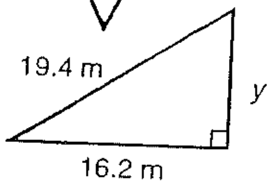
(b)



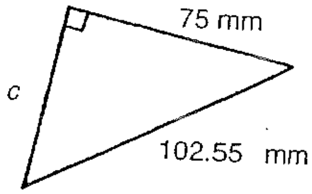
(c)



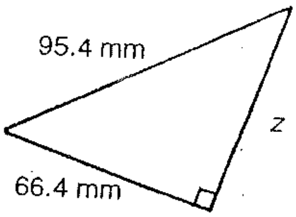
(d)



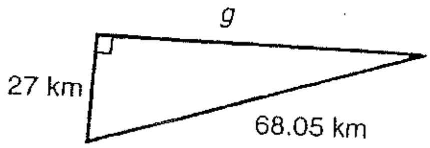
(e)



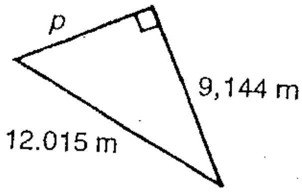
(f)



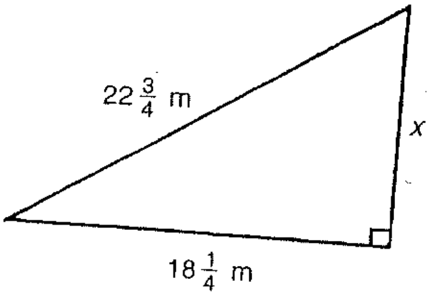
(g)



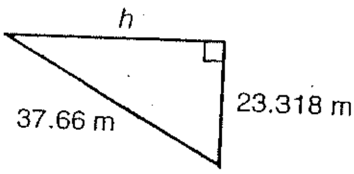
(h)



(i)



(j)



2. Find the length of the third side of a right-angled triangle with hypotenuse and a smaller side with these measurements:

(a) 104 cm, 96 cm

(b) 168 mm, 175 mm

(c) 0.34 m, 0.3 m

(d) 0.5 cm, 0.48 cm

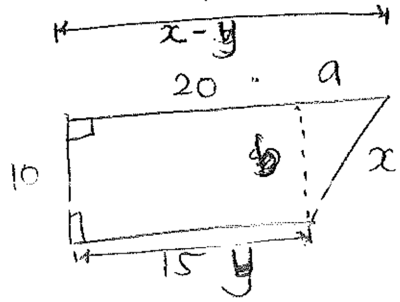
(e) 1 km, $\frac{3}{5}$ km

(f) $\frac{1}{4}$ km, 2 km

(g) 22.5 cm, 6.3 cm

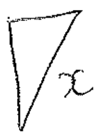
(h) 35.1 cm, 13.5 cm

M] Example

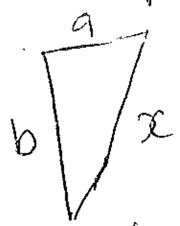


Steps

1. Draw you right angle triangle



2. Label you right angle triangle



3. To find the length of a

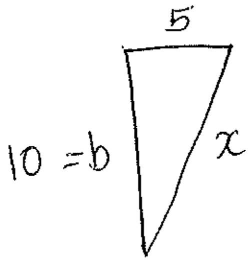
$a = 20 - 15 = 5$

$a = 5$

4. The height of the right angle

$b = 10$

5. Redraw your △



$c^2 = a^2 + b^2$

$x^2 = 5^2 + 10^2$

$x^2 = 25 + 100$

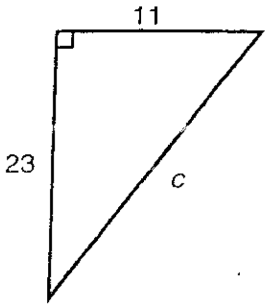
$\sqrt{x^2} = \sqrt{125}$

$x = 11.18$

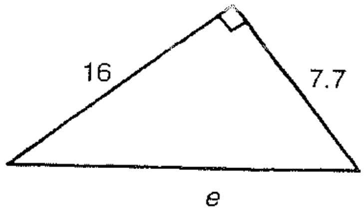
* Inverse of Square is Square root.

* Give your answers to two decimal places where necessary.
* units of measurement in centimeters (cm).

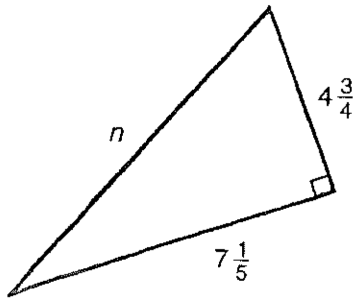
(i)



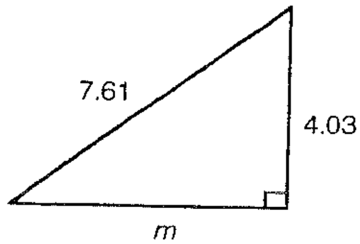
(j)



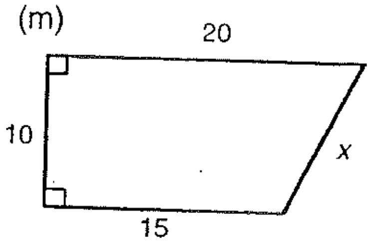
(k)



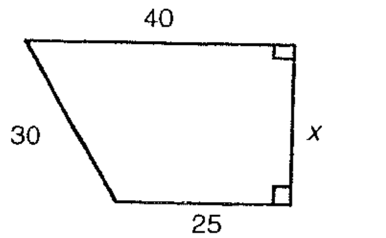
(l)



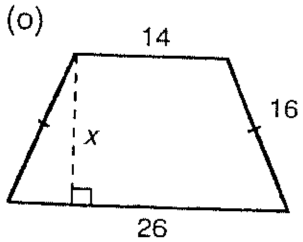
(m)



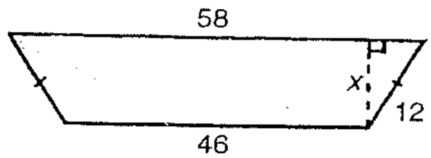
(n)



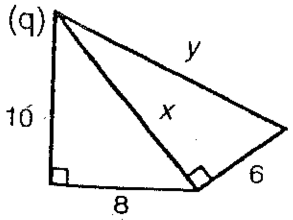
(o)



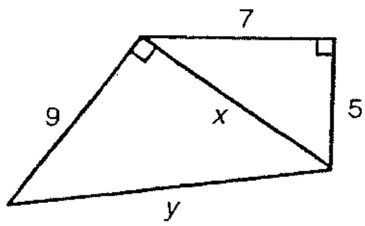
(p)



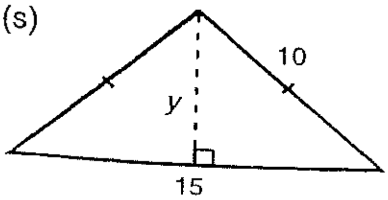
(q)



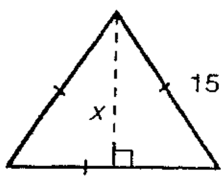
(r)



(s)



(t)

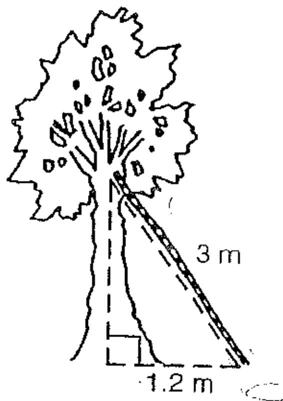


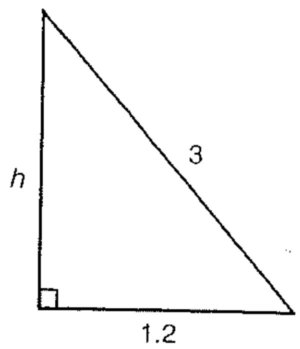
Applications of Pythagoras's Theorem

Right-angled triangles are very common shapes in both natural and man-made situations. We often need to find a particular side of a triangle and can use Pythagoras's Theorem to solve the problem.

Example 1 A 3 metre long ladder is leaning against a tree. The base of the ladder is 1.2 metres from the bottom of the tree. How high up the tree does the ladder reach?

Important !!
Always draw a diagram.



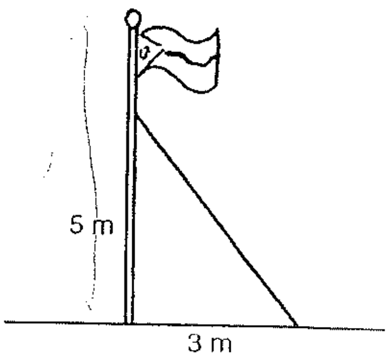


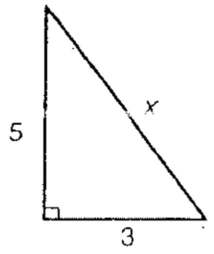
Using Pythagoras's Theorem:

$$c^2 = a^2 + b^2$$
$$3^2 = h^2 + 1.2^2$$
$$9 = h^2 + 1.44$$
$$9 - 1.44 = h^2$$
$$7.56 = h^2$$
$$\sqrt{7.56} = \sqrt{h^2}$$
$$2.75 = h$$

∴ The ladder reaches 2.75 m up the tree.

Example 2 A support wire is attached 5 metres up flagpole. The other end is attached to the ground 3 metres from the base of the flagpole. How long is the wire?





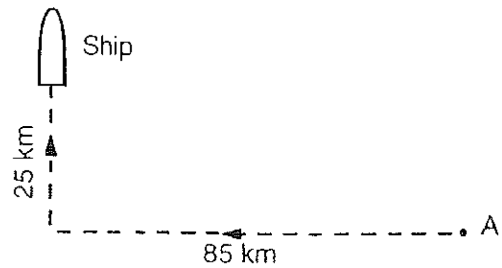
Using Pythagoras's Theorem:

$$c^2 = a^2 + b^2$$
$$x^2 = 5^2 + 3^2$$
$$= 25 + 9$$
$$= 34$$
$$\sqrt{x^2} = \sqrt{34}$$
$$x = 5.83$$

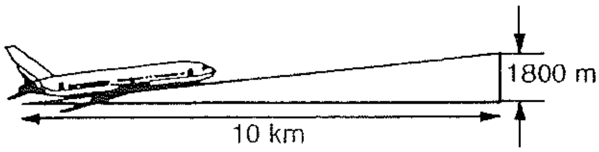
The wire is 5.83 metres long.

Exercise 4.5

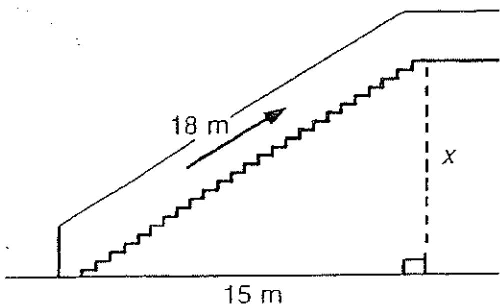
1. A ship travels 85 km due west and then 25 km due north. How many km is it now in a direct line from its starting point A? (Give answer correct to 3 significant figures.)



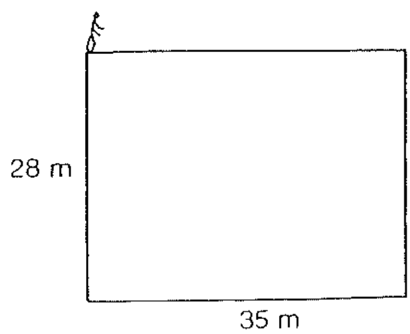
2. An aeroplane on a straight take-off path reaches a height of 1800 m after covering a distance of 10 km on the ground. What is the length of its take-off path to the nearest 0.1 km?



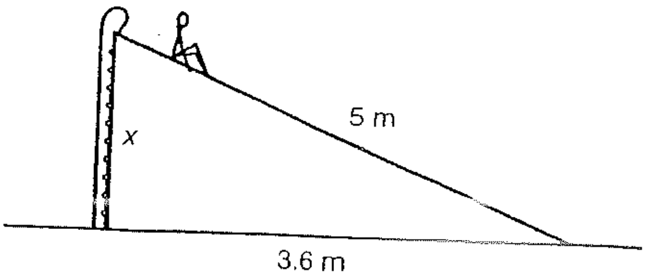
3. The steps going up to a house are 18 m long. When you walk up the steps, you move across 15 m. How high up have you travelled?



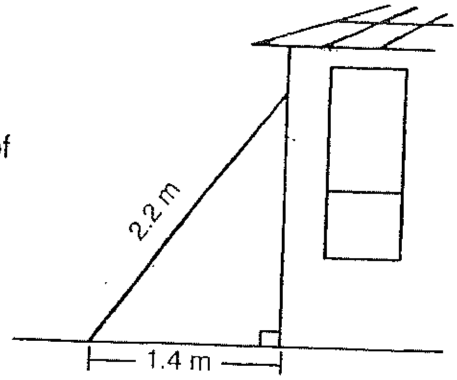
4. Andrew is standing at the corner of the park, and has decided to cross diagonally rather than go around.
   (a) How far does Andrew travel when crossing the park?
   (b) How much distance does he save by not going around?



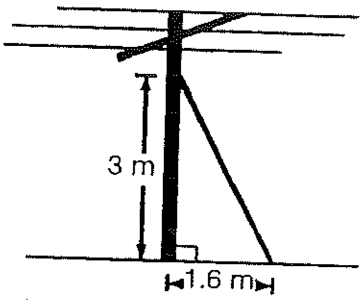
5. The bottom of a slide is 3.6 m from the base of the vertical ladder. If the slide is 5 m long, how high is the ladder?



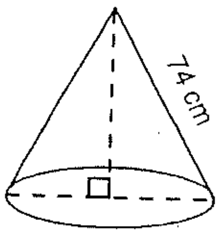
6. A ladder 2.2 m long leans against the side of a building with its base 1.4 m from the building. How far up the side of the building does the ladder reach? (Give your answer correct to 2 decimal places)



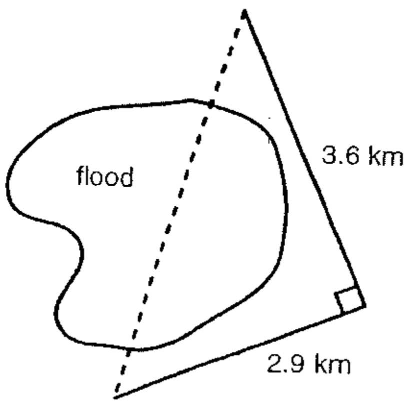
7. Power poles sometimes need to be supported up right by a cable fixed at ground level. What length of cable is required to support this pole?



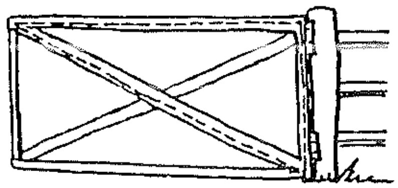
8. Calculate the vertical height of the cone shown. The diameter of its base is 56 cm. (Give answer correct to the nearest cm.)



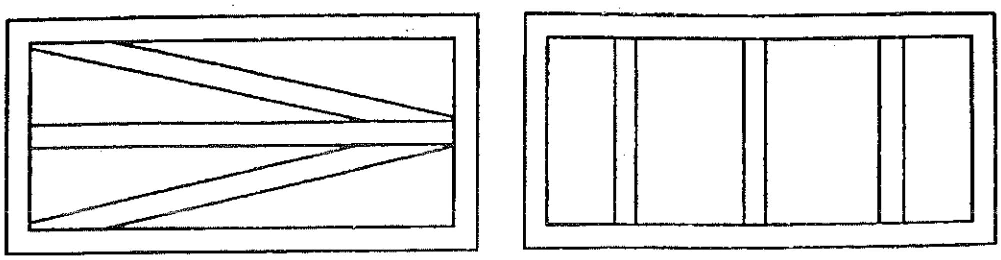
9. During a heavy rainstorm, the road that Alison normally takes to travel home from school has become flooded. She must take a detour as shown in the diagram. How much further does she now have to travel?



10. Angela has built a new rectangular gate from timber. The gate is 1.5 m high and 3.1 m wide. She has decided to put in two diagonal cross supports to give the gate more strength. What is the total length of extra wood she will need?



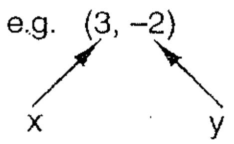
The cost of the gate depends on the amount of timber it takes to make it. Would it be cheaper to make the gate from one of these designs?

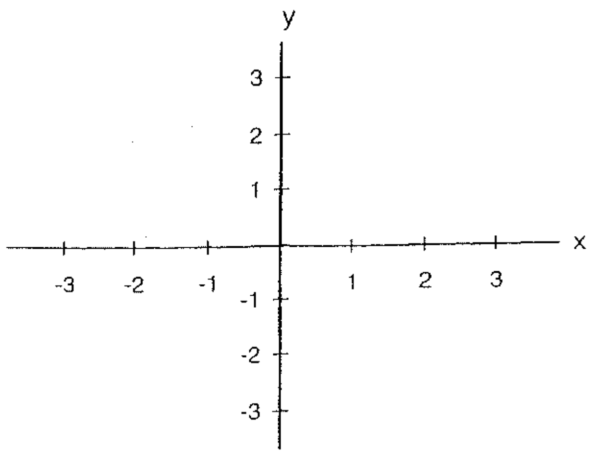


Pythagoras and Co-ordinate Geometry

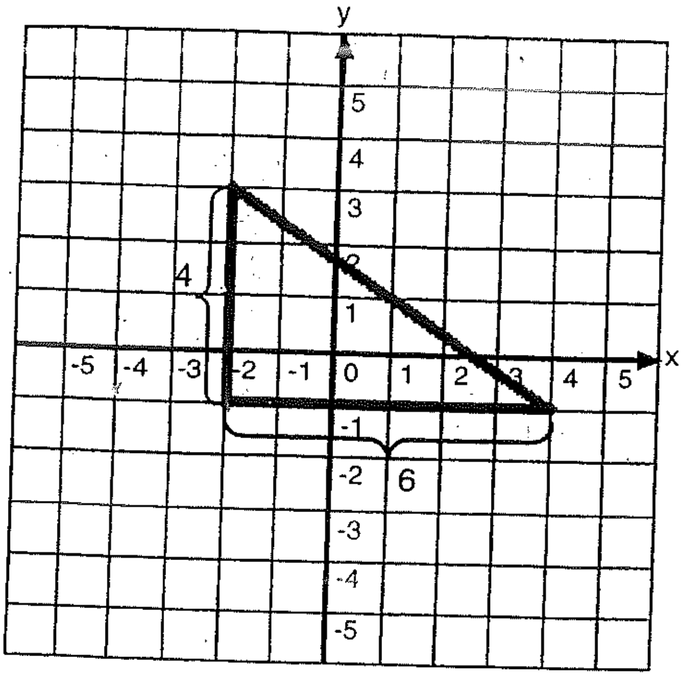
We can use Pythagoras's Theorem to find the distance between two points.

Points (or co-ordinates) need to be plotted first.





Example: Find the distance between (−2, 3) and (4, −1)



After plotting the points, join them to form a right-angled triangle.

The distance between the points is equal to the hypotenuse of the triangle.

$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
&= 4^2 + 6^2 \\
&= 16 + 36 \\
&= 52 \\
\sqrt{c^2} &= \sqrt{52} \\
c &= 7.211 \text{ units.}
\end{aligned}
$$

Exercise 4.6

1. Find the distance between each pair of points:

(a) (2,5) and (−1,1)

(b) (−3,−2) and (2,3)

(c) (−4,1) and (2,−1)

(d) (2,1) and (−5,2)

(e) (−3,0) and (3,3)

(f) (3,−1) and (−1,3)

(g) (0,−2) and (−3,4)

(h) (−2,−5) and (0,5)

(i) (5,−1) and (1,5)

(j) (1,−2) and (6,2)

(k) (−2,−4) and (−1,5)

(l) (−5,−1) and (0,6)

Pythagorean Triples

A Pythagorean Triple is a group of three whole numbers that satisfy Pythagoras's Theorem.

The simplest group is 3, 4, 5.

$$3^2 + 4^2 = 5^2$$

The most common triples are:

3, 4, 5	==>	$3^2 + 4^2 = 5^2$	==>	$9 + 16 = 25$
5, 12, 13	==>	$5^2 + 12^2 = 13^2$	==>	$25 + 144 = 169$
7, 24, 25	==>	$7^2 + 24^2 = 25^2$	==>	$49 + 576 = 625$
8, 15, 17	==>	$8^2 + 15^2 = 17^2$	==>	$64 + 225 = 289$

If we multiply these triples by a certain number we can got more triples.

(3, 4, 5)	x 2	==>	(6, 8, 10)
	x 5	==>	(15, 20, 25)
	x $\frac{1}{2}$	==>	($1\frac{1}{2}$, 2, $2\frac{1}{2}$)
	etc.		

Exercise 4.7

1. Test the following groups of numbers to find out which ones are Pythagorean Triples.

   (a) 10, 24, 26
   (b) 30, 40, 50
   (c) 9, 13, 21
   (d) 5, 12, 13
   (e) 9, 12, 15
   (f) 16, 30, 34
   (g) 2.5, 6, 6.5
   (h) 3.5, 12, 12.5
   (i) 18, 24, 30
   (j) 10, 12, 14

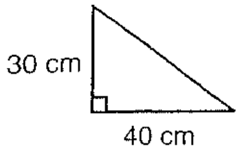
2. Using the four common triples given below, copy and complete the multiple triples:

   Use (3, 4, 5), (5, 12, 13)
   (8, 15, 17), (7, 24, 25)

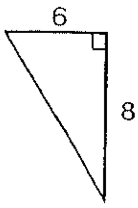
   (a) (12, 16, ...)
   (b) (4, 7.5, ...)
   (c) (25, 60, ...)
   (d) (21, 72, ...)
   (e) (16, 30, ...)
   (f) (80, 150, ...)
   (g) (10.5, 36, ...)
   (h) (4.5, 6, ...)
   (i) (........., 48, 52)
   (j) (9, ..., 15)
   (k) (3.5, ..., 12.5)
   (l) (........., 45, 51)

3. Without doing any calculations (use the triples), write down the size of the unknown sides:

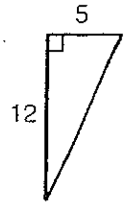
(a)



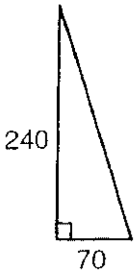
(b)



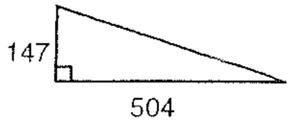
(c)



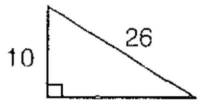
(d)



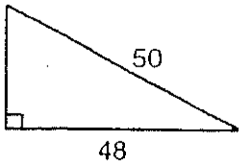
(e)



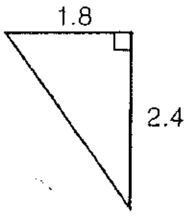
(f)



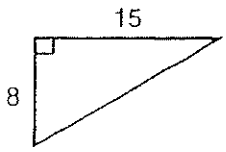
(g)



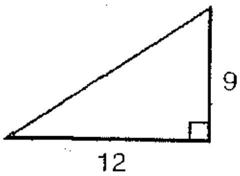
(h)



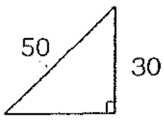
(i)



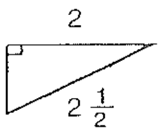
(j)



(k)



(l)

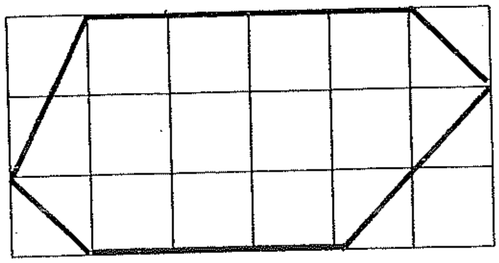


Topic 3 – Area , Surface Area, and Volume

The area of a figure is the measure of the region inside the figure.

The area can be found by counting the number of squares which will fit into a figure.

For example,

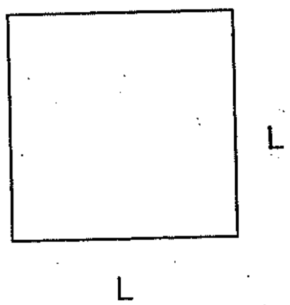


Area = 14 squares

However, we prefer to use a formula to work out the area because it will give us an accurate answer.

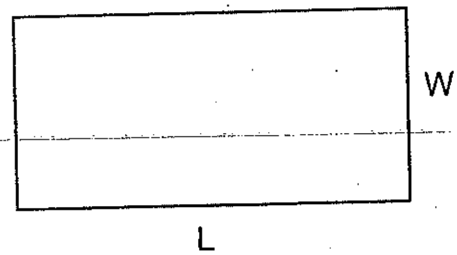
Area Formulae Summary

1. **Square:**



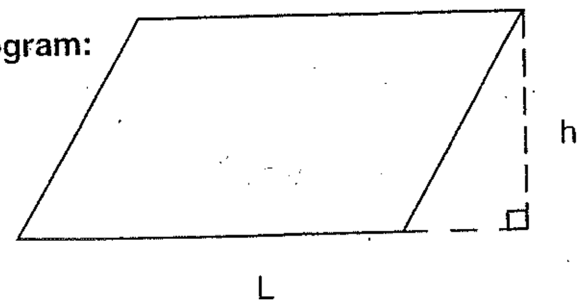
Area = Length x width
= L x L
= L^2

2. **Rectangle:**



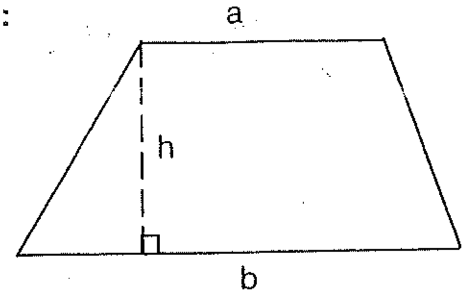
Area = length x width
= L x W

3. **Parallelogram:**



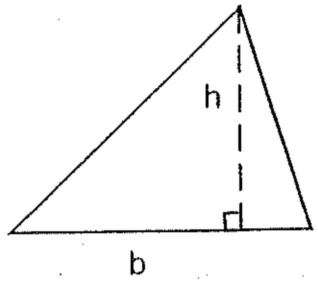
Area = length x height
= L x h

4 **Trapezium:**



$$\text{Area} = \frac{1}{2}(a + b)\,h$$

5. **Triangle:**



$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2}\ b \times h$$

Examples Find the areas of the following:

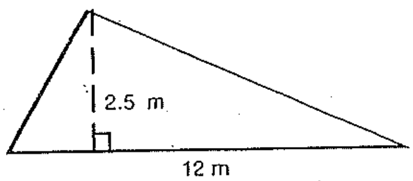
a.

$$\text{Area} = L \times W$$

$$= 9 \times 3$$

$$= 27\ \text{cm}^2$$

b.

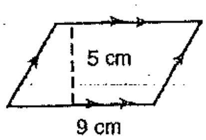


$$\text{Area} = \frac{1}{2}\, b \times h$$

$$= \frac{1}{2} \times 12 \times 2.5$$

$$= 15\ \text{m}^2$$

c.

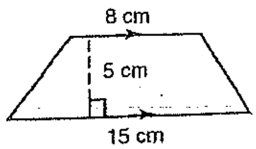


$$\text{Area} = L \times h$$

$$= 9 \times 5$$

$$= 45\ \text{cm}^2$$

d.



$$\text{Area} = \frac{1}{2}(a + b)\,h$$

$$= \frac{1}{2}(8 + 15)\,5$$

$$= \frac{1}{2} \times 23 \times 5$$

$$= 57.5\ \text{cm}^2$$

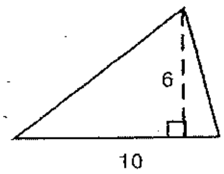
Exercise 3.1

1. Find the area of:

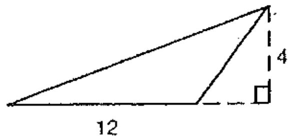
    a. a square of side 3cm
    b. a square of side 7cm
    c. a square of side 4.7m
    d. a square of side 8.9mm
    e. a rectangle 6cm by 7.7cm
    f. a rectangle 9m by 4m
    g. a rectangle 6.5cm by 7.5cm
    h. a rectangle 4.6m by 3.9m
    i. a rectangle 1.2cm by 8mm
    j. a rectangle 25mm by 0.2m

2. Find the area of the following shapes: (all measurements in cm)

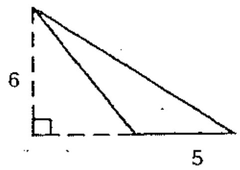
a.



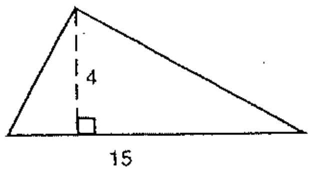
b.



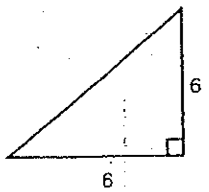
c.



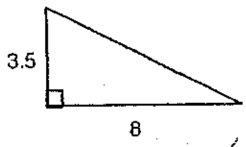
d.



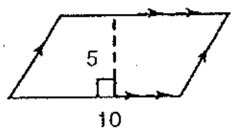
e.



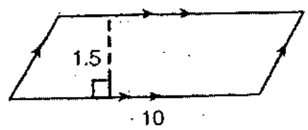
f



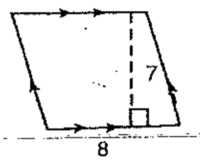
g.



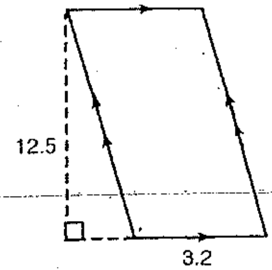
h.



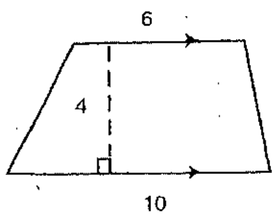
i.



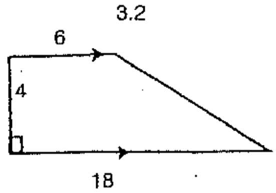
j.



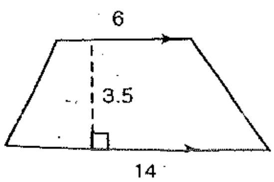
k.



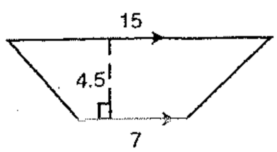
l.



m.

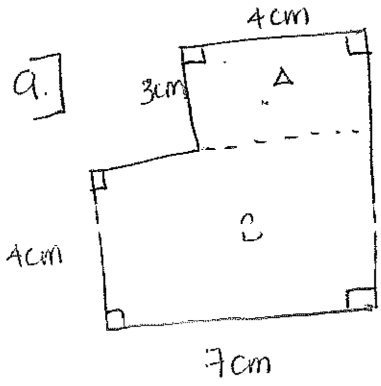


n.



Example for Question 3

a.]



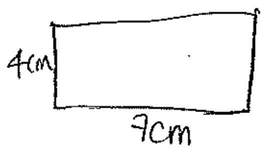
Shape A

* Rectangle:
* A = L x W

A = 4 x 3

A = 12 cm²

Shape B



* Rectangle
* A = L x W

A = 4 x 7

A = 28 cm²

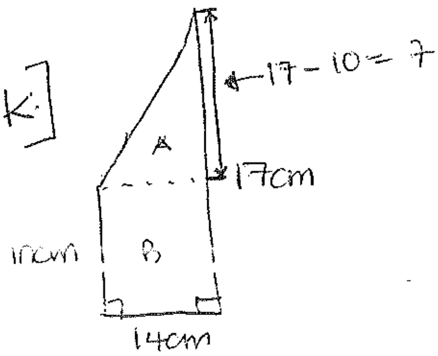
Since it is a combined figure.

A = SA + SB

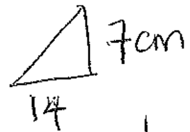
= 12cm² + 28cm²

A = 40 cm².

k.]



Shape A



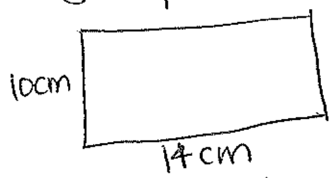
* Triangle
* ½ x b x H

A ½ x 14 x 7

A = 7 x 7

A = 49cm²

Shape B



* Rectangle
* L x W

A = 14 x 10

A = 140 cm²

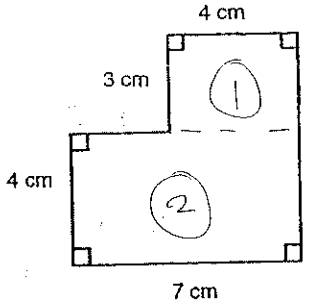
T/A = SA + SB

T/A = 49cm² + 140cm²

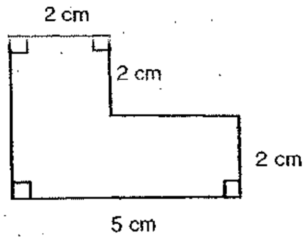
T/A = 189 cm²

3. Find the area of the following combined figures:

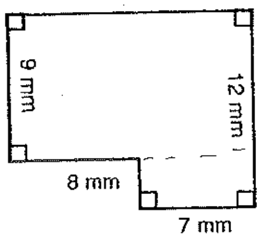
a.



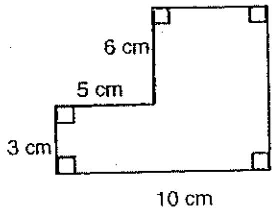
b.



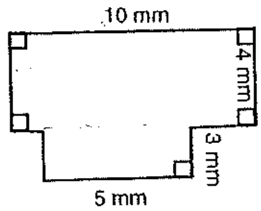
c.



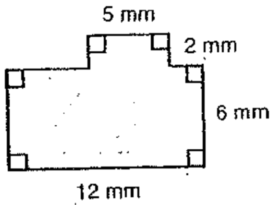
d.



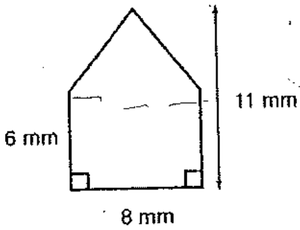
e.



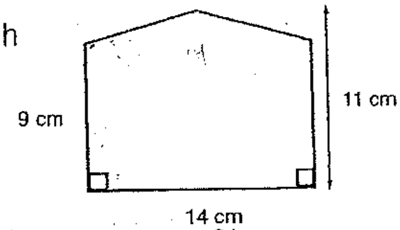
f..



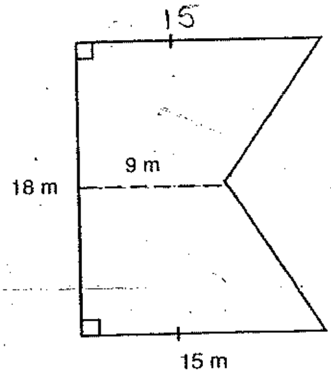
g.



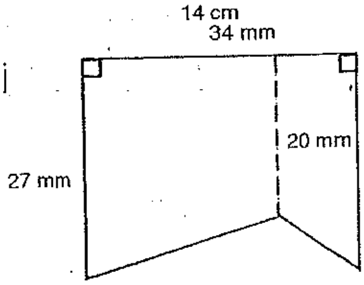
h



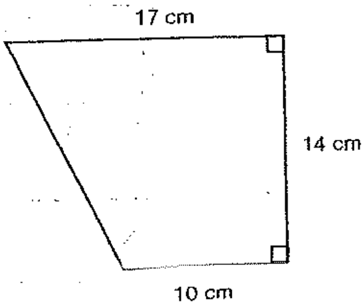
i.



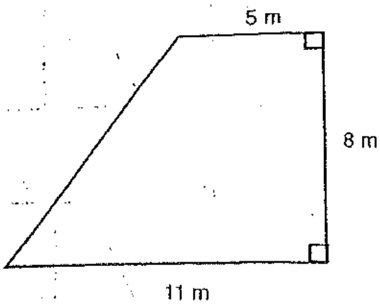
j



k.



l.

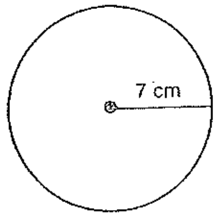


Area of Circles

The formula for the area of a circle is:

Area	=	πr^2	Where $\pi \approx 3.14$ (2 d.p.) and r is the radius.

Example 1 Find the area of the circle shown:



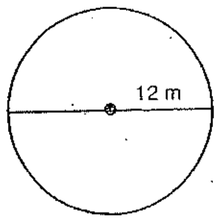
Area $= \pi r^2$

$= 3.14 \times 7^2$

$= 3.14 \times 49$

$= 153.86 \text{ cm}^2$

Example 2 Find the area of the circle shown:



The diameter is 12m.

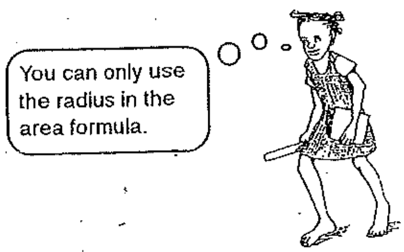
So the radius is 6m.

Area $= \pi r^2$

$= 3.14 \times 6^2$

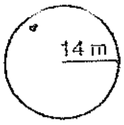
$= 3.14 \times 36$

$= 113.04 \text{m}^2$

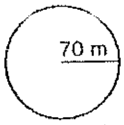


Exercise 3.2

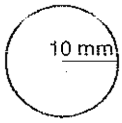
1.



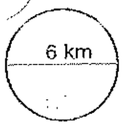
2.



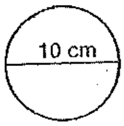
3.



4.



5.



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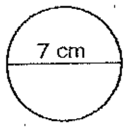

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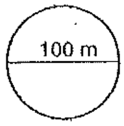

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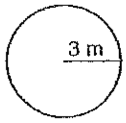

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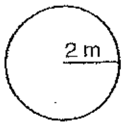

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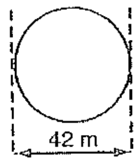

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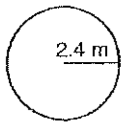

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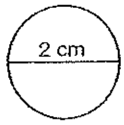

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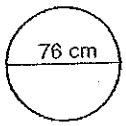

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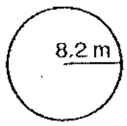

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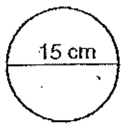

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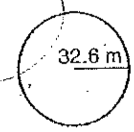
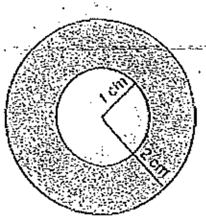


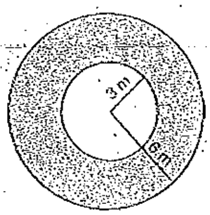
17. Find the answer to each of the following, including a diagram for each question:
    a. A compact disc has a radius of 8.7cm. Find the area of a compact disc.
    b. Find the area of a circular rug with a diameter of 3.05m.
    c. A coin has a diameter of 2.2cm. Find its area.
    d. Find the area of the base of a candle if its radius is 13mm.

18. An annulus is a donut shape. Find the area of the following shapes:

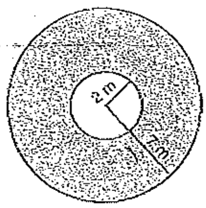
a.



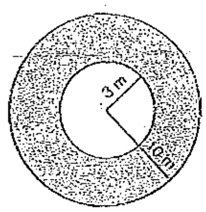
b.



c.



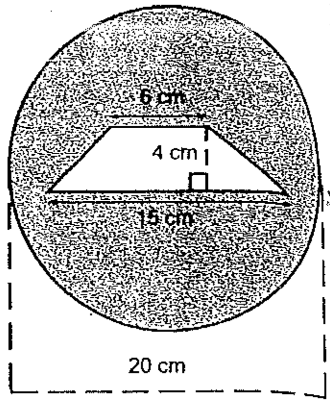
d.



Combined Shapes

Example 1. Find the shaded area of:

The figure is a circle of **radius** 10cm with a trapezium cut out of it.



Circle area $= \pi r^2$

$= 3.14 \times 10^2$

$= 314cm^2$

Trapezium area $= \frac{1}{2}(a + b)h$

$= \frac{1}{2}(6 + 15) \times 4$

$= \frac{1}{2} \times 21 \times 4$

$= 42cm^2$

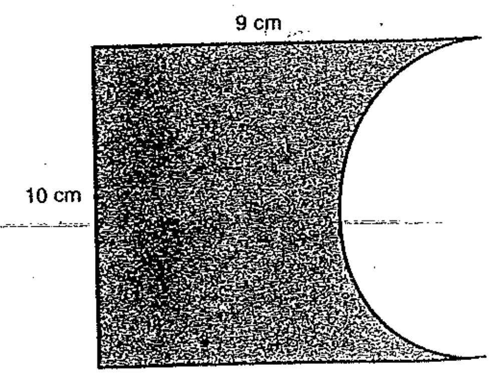
Shaded area = Circle area - Trapezium area

$= 314 - 42$

$= 272cm^2$

Example 2. Find the area of the figure:

The diagram shows a rectangle with a semi-circle cut out of it.



Rectangular area $= L \times W$

$= 10 \times 9$

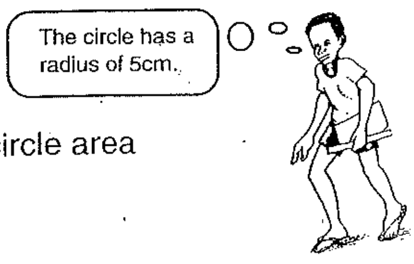
$= 90cm^2$

Semicircle area = half of πr^2

$= \frac{1}{2} \times \pi \times 5^2$

$= \frac{1}{2} \times 3.14 \times 25$

$= 39.25cm^2$



Area of figure = Rectangular area - Sem-icircle area

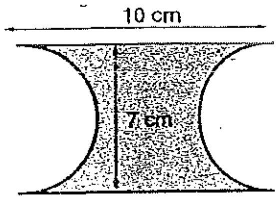
$= 90 - 39.25$

$= 50.75cm^2$

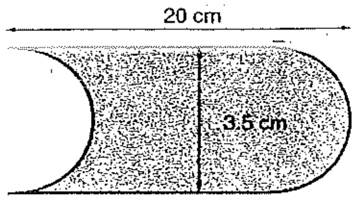
Exercise 3.3

1. Find the area of the following:

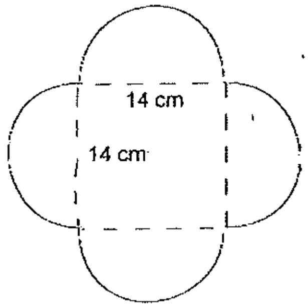
a.



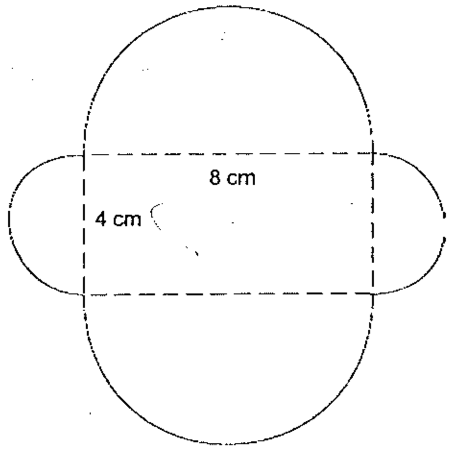
b.



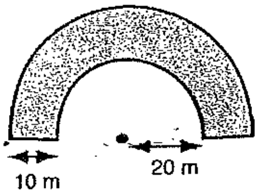
c.



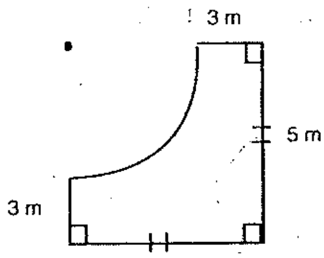
d.



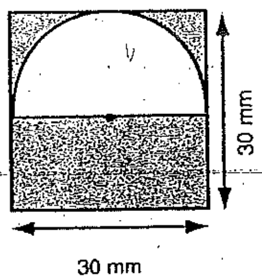
e.



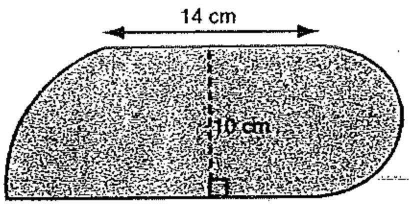
f.



g.

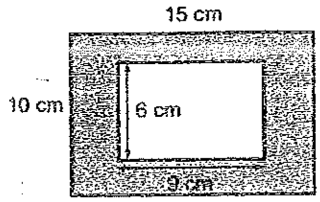


h.

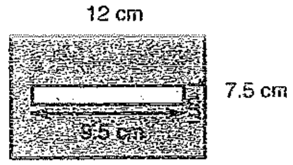


2. Find the area of the shaded portion of the following figures:

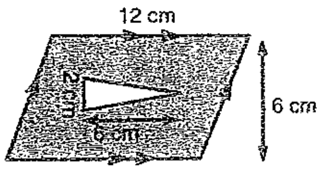
a.



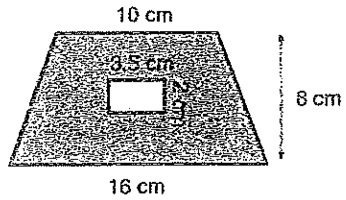
b.



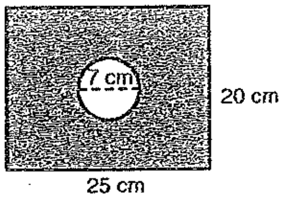
c.



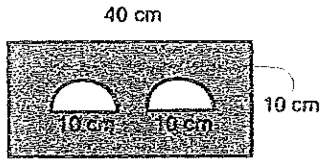
d.



e.

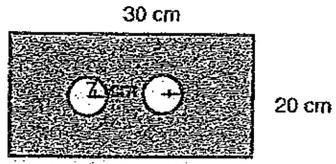


f.

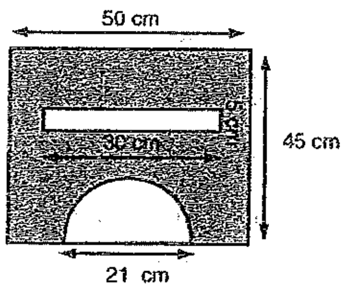


3. Find the shaded area:

a.



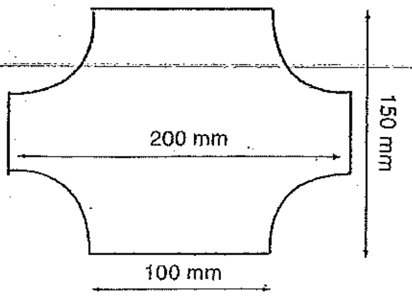
b.



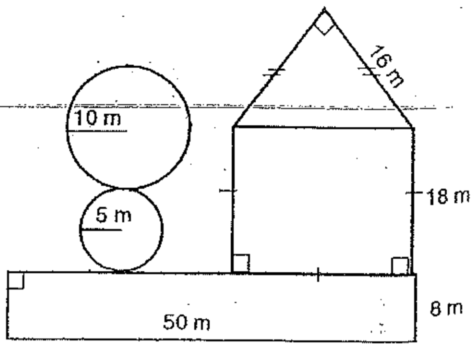
Bank teller's window

4. Find the area of the following figures:

a.



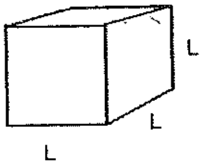
b.



Surface Area

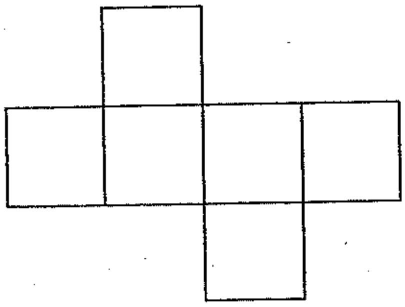
If we painted the outside of a three dimensional shape we would be painting what is called the **surface area** of that shape. It is useful to know how different shapes are made.

CUBE



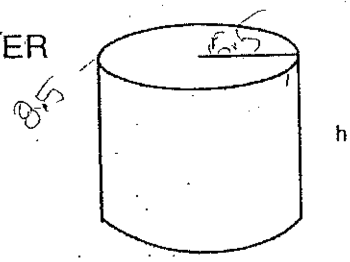
= 6 squares folded together

=



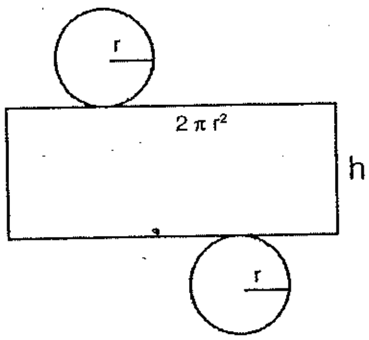
Surface area = $6 \times L^2$

CYLINDER



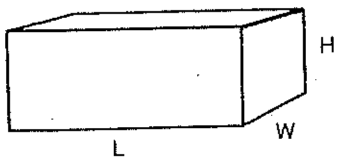
= a rectangle rolled up with a circle at each end.

=



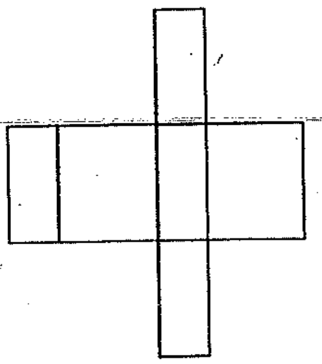
Surface area = $2\pi r^2 + 2\pi r h$

CUBOID



= a box shape made up of rectangles.

=



Surface area = 2 LW + 2 WH + 2 LH

Examples:

1. Find the surface area of a cube with side length 8cm.

Area of one square = L^2

= 8×8

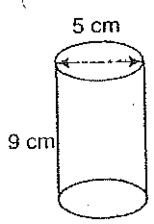
= 64 cm^2

Surface area = 6 x area of square

= 6×64

= 384 cm^2

2. Find the surface area of a cylinder



Area of a circle = πr^2

= $\pi \times 2.5^2$

= 19.63 cm^2

Area of rectangle = $2\pi r \times h$

= $2 \times \pi \times 2.5 \times 9$

= 141.37 cm^2

Surface area = area of rectangle + area of two circles.

= $141.37 + 2 \times 19.63$

= 180.63cm^2

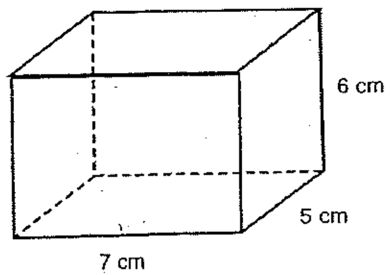
3. Find the surface area of a cuboid of length 7cm, width 5cm and height 6cm.

Surface area = 2 x front + 2 x side + 2 x top

= $2 \times 7 \times 6 + 2 \times 5 \times 6 + 2 \times 7 \times 5$

= $84 + 60 + 70$

= 214 cm^2

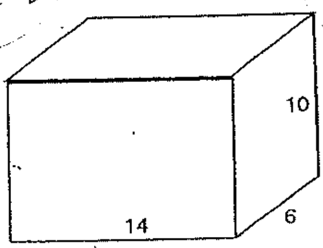


Exercise 3.5

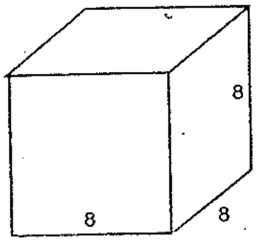
1. Find the surface area of:
   a. a cube of side 7cm
   b. a cube of side 3.2cm
   c. a cuboid of side 8cm by 5cm by 3cm
   d. a cuboid of side 2.5cm by 2.5cm by 5cm
   e. the **curved** surface of a cylinder with radius 7cm and height 10cm
   f. the **curved** surface of a cylinder with diameter 11cm and height 12cm
   g. the surface area of a cylinder with radius 5cm and height 22cm
   h. the surface area of a cylinder with diameter 6.5cm and height 4.1cm

2. Find the surface area of these solids. (All measurements are in centimetres.)

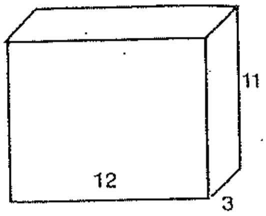
a.



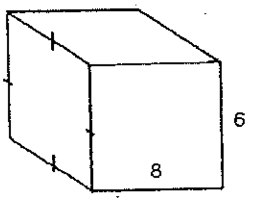
b.



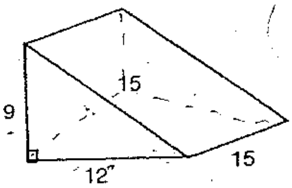
c.



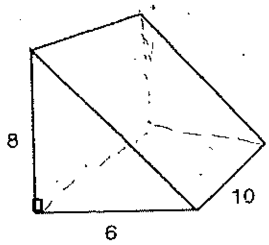
d.



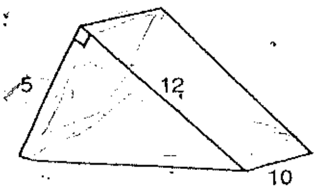
e.



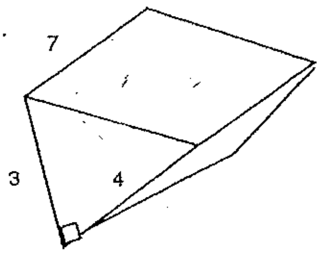
f.



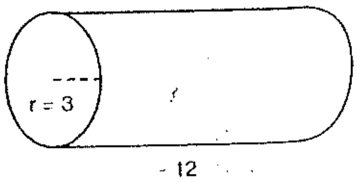
g.



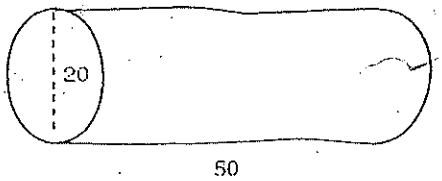
h.



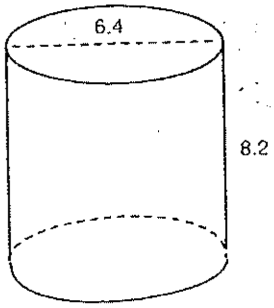
i.



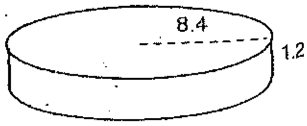
j.



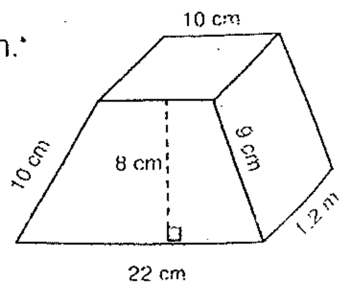
k.



l.

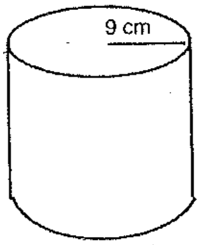


m.

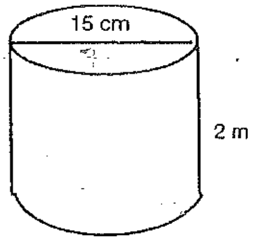


3. Find the total surface area of the following closed cylinders by calculating:
   i. areas of the two ends
   ii. area of the curved surface
   iii. the total surface area.

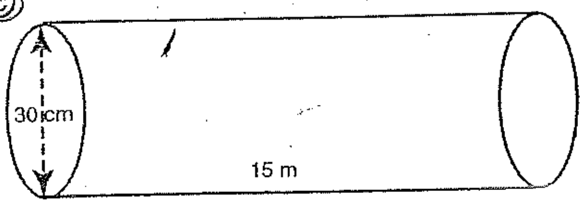
a.



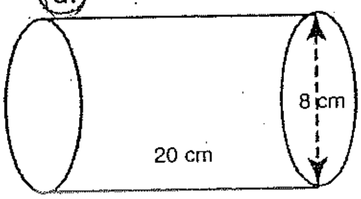
b.



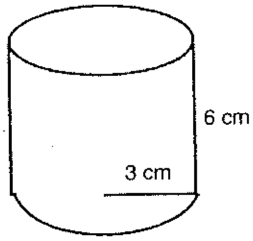
c.



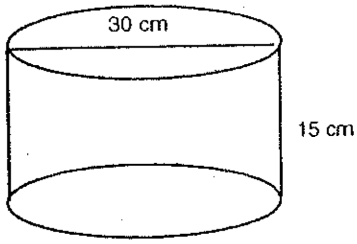
d.



e.

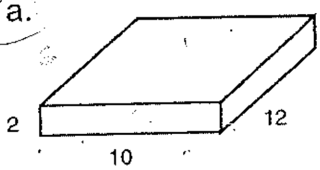


f.

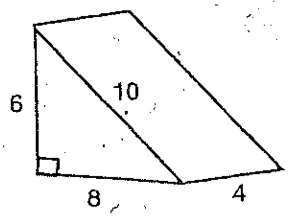


4. Find the surface area of:

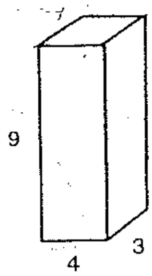
a.



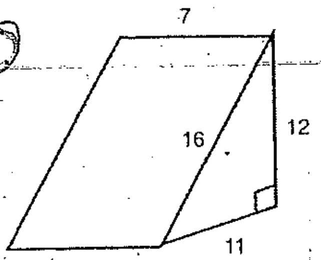
b.



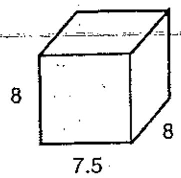
c.



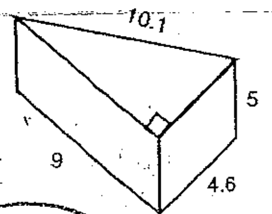
d.



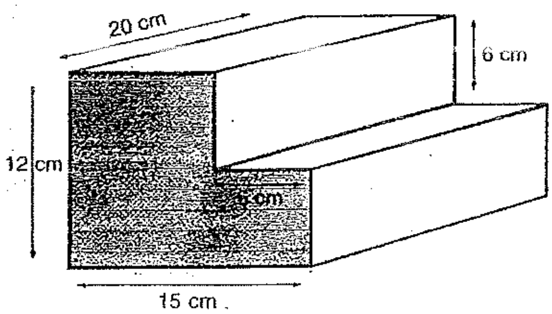
e.



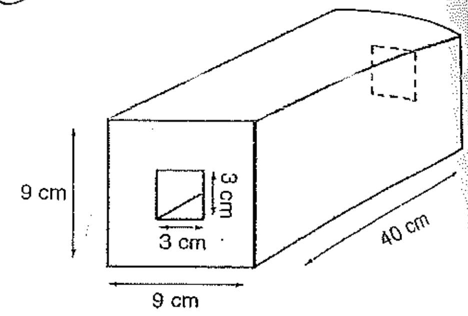
f.



5. a.

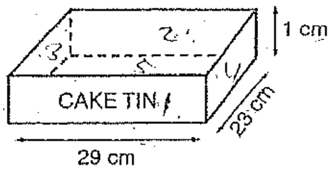


b.

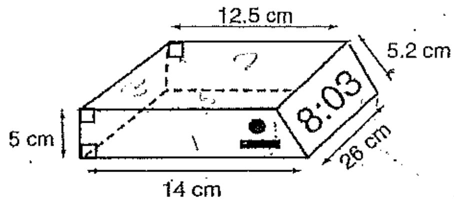


Exercise 3.6

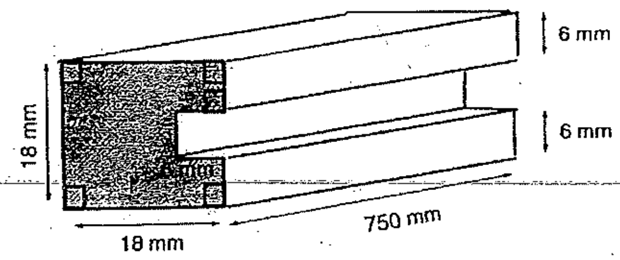
1. Find the surface area of an **open** rectangular cake tin (outside surfaces only).



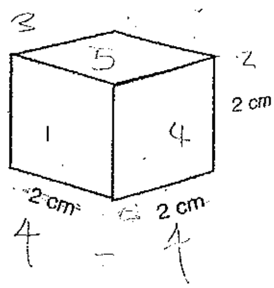
2. Find the surface area of a clock radio.



3. Find the surface area of a grooved block.

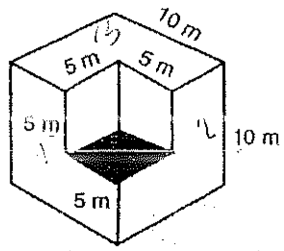


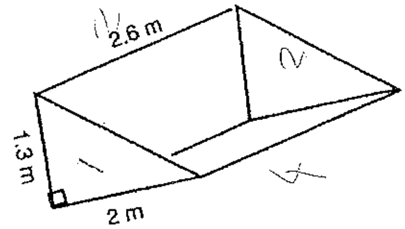
4. This cube has 6 faces. The length of each side is 2cm.
   a. Calculate the surface area of the cube.
   b. If the length of each side is doubled, calculate the new surface area.



Exercise 3.6 (continued)

5. A cube is made out of 4 smaller cubes. One small cube is lost, as shown in the diagram. Find the surface area of the solid.





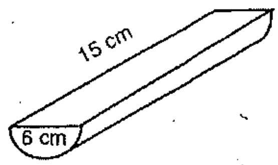
6. The "scoop" on the front end loader is in the shape of a triangular prism with one rectangle removed. What area of metal is needed to make the scoop?

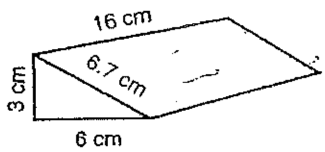
7. Find the total surface area (inside and outside) of a pipe 10 metres long and 0.25 metre radius (no ends).

8. If 1 litre of paint covers $15m^2$, how many litres are needed to paint the curved surface and top of a cylinder 8.5m high and 6.5m in diameter?

9. How many square metres of cardboard are needed to make 100 boxes measuring 20cm x 10cm x 31cm?

10. A manufacturer is thinking of making bars of chocolate in either one of these two shapes, and wrapping them in expensive wrappers.





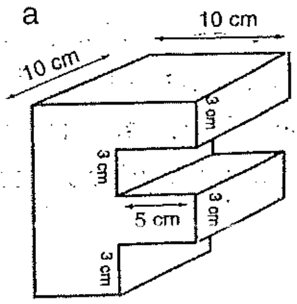
Which shape would be the least expensive to wrap?

11. A box with an open top is 410mm high. The base measures 595mm by 600mm. Calculate the external surface area of the box in square metres.

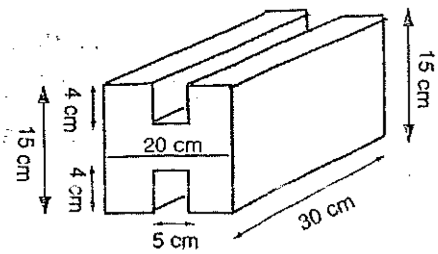
12. Find the area of the curved surface of a cylinder with diameter 5m and height 255cm.

13. David wants to paint a 5m x 3.5m x 4m cuboid red with paint which covers $14m^2$ per litre. How many 4 litre tins of paint will be needed if the cuboid is given two coats of red paint?

14. Find the surface area of: a



b.

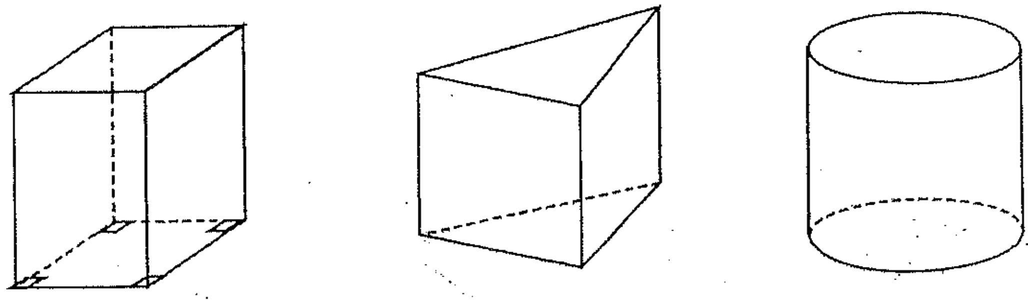


Volume

The volume of a 3 dimensional shape is the amount of space inside it. We measure volume in **cubic units**.

Name	Symbol	The same as
1 cubic metre	$1m^3$	$100cm \times 100cm \times 100cm = 1,000,000cm^3$
1 cubic centimetre	$1cm^3$	$100mm \times 100mm \times 100mm = 1000mm^3$
1 cubic millimetre	$1mm^3$	

A **prism** is a 3 dimensional shape which has a constant cross-sectional area. This means that when a prism sits on one end, the top is the same shape as the base.



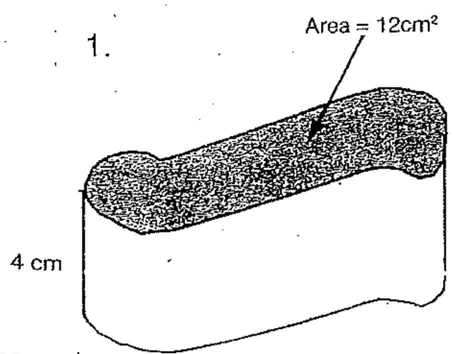
Rectangular prism (cuboid) | Triangular prism | Circular prism (cylinder)

The volume of any prism is given by the formula:

Volume of prism = area of base x height

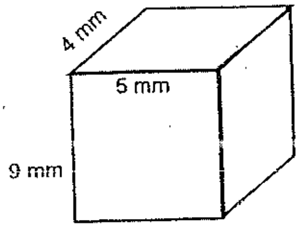
Examples Find the volumes of the following prisms:

1.



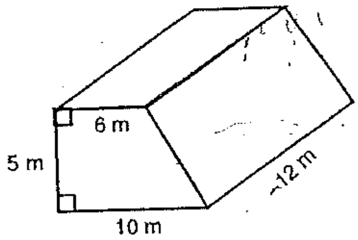
Volume = Area of base x height
= 12 x 4
= $48cm^3$

2.



Area of base = 5×4
= $20mm^2$
Volume = 20×9
= $180mm^3$

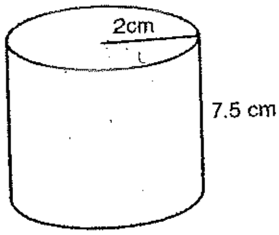
3.



Area of trapezium = $\frac{1}{2}(a+b)h$
= $\frac{1}{2}(6+10) \times 5$
= $40m^2$

Volume = 40×12
= $480m^3$

4.



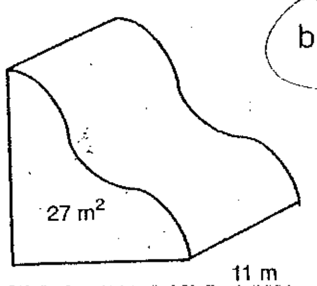
Area of circle = πr^2
= $\pi 2^2$
= $12.566cm^2$

Volume = 12.566×7.5
= $94.245cm^3$

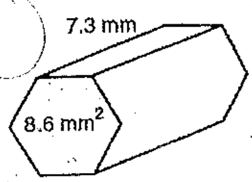
Exercise 3.7

1. Find the volumes of the following prisms. Give your answers to two decimal places where necessary.

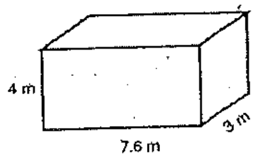
a.



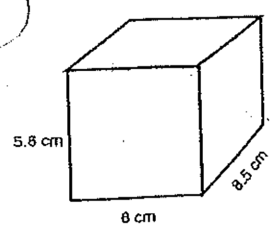
b.



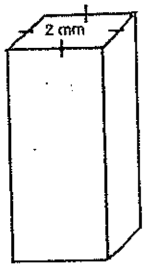
c.



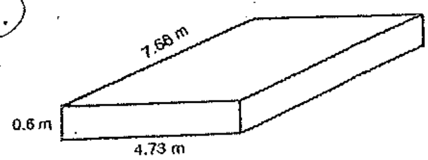
d.



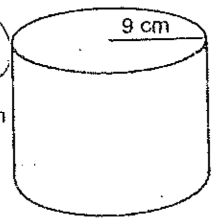
e.



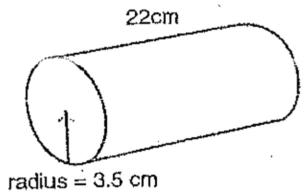
f.



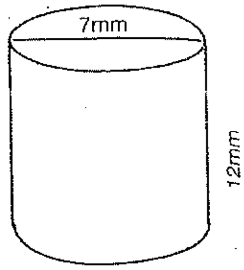
g.



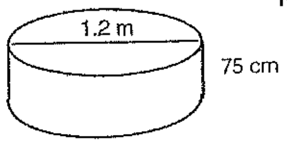
h.



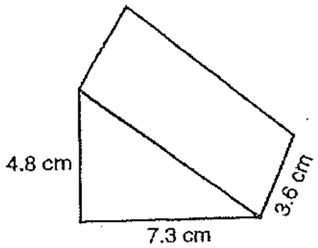
i.



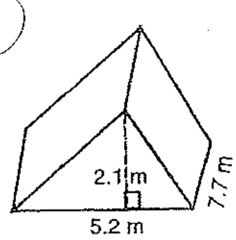
j.



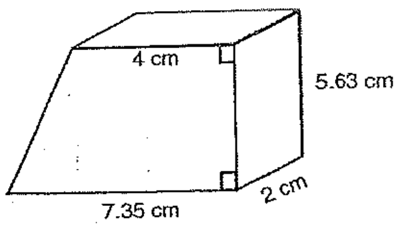
k.



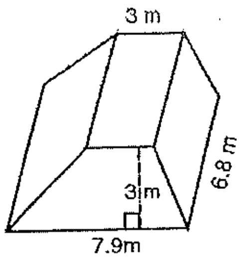
l.



m.



n.



2. Find the volume of the following cylinders, giving your answers to one decimal place.
   a. radius of base 10.5cm, height 0.6m
   b. diameter of top 8cm, height 14cm
   c. diameter of base 0.75m, length 60cm
   d. radius of base 1.4cm, height 63mm
   e. radius of top 7mm, height 1.3m (answer in cm^3)

Volume in litres

When a container is filled with liquid, its volume is measured in litres (L) or millilitres (mL).

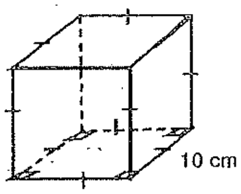
The connection between cm^3 and mL is a simple one:

1mL = 1cm³

That means a cube with 1cm edges will hold exactly 1mL of liquid.

For example, a cube with edge 10cm

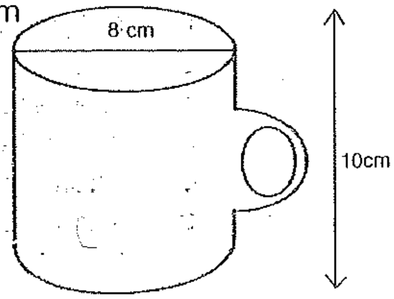
$$\begin{aligned} \text{Volume} &= 10cm \times 10cm \times 10cm \\ &= 1000cm^3 \\ &= 1000mL \\ &= 1L \end{aligned}$$



Example Find the volume of a cylindrical cup with diameter 8cm and height 10cm. Give your answer to the nearest 10mL..

Radius = 4cm, Height = 10cm

$$\begin{aligned} \text{Volume} &= \pi \times 4^2 \times 10 \\ &= 502.4cm^3 \\ &= 502.4mL \\ &\approx 500mL \text{ (to the nearest 10mL)} \end{aligned}$$



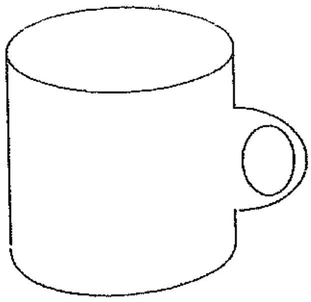
Exercise 3.8

Calculate the volume of each container to the nearest 10mL

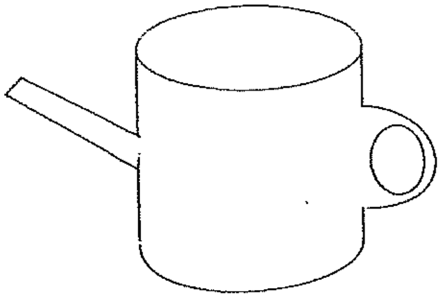
1. Rectangular boxes with the following dimensions:
   a. 16cm, 7.5cm, 1cm
   b. 25cm, 25cm, 25cm
   c. 15cm, 15cm, 28cm
   d. 31cm 15.9cm, 17.6cm

2. Cylindrical cups
   a. radius = 2.7cm, height = 14cm
   b. diameter = 7.1cm, height = 5.5cm
   c. radius = 14mm, height = 3cm

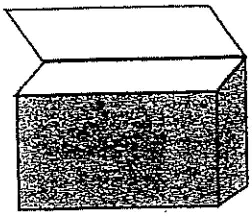
3. Mug: diameter = 6.7cm
height = 5.9cm



4. Watering can: radius = 9.7cm
height = 20cm



5. Box: length = 12cm
width = 5cm
height = 7cm

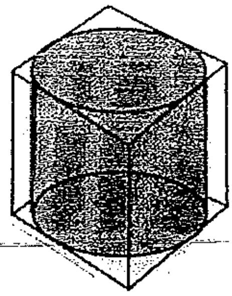


6. The petrol tank of a small car is cylindrical with a diameter of 30cm and length of 75cm. Find the capacity of the tank (how many litres it holds) to the nearest litre.

7. A water tank has a radius of 125cm and a height of 2.1m. How many litres will the tank contain?

8. Calculate the weight of a cylindrical iron bar of radius 12mm and length 22cm if 1cm^3 of the iron bar weighs approximately 8 grams.

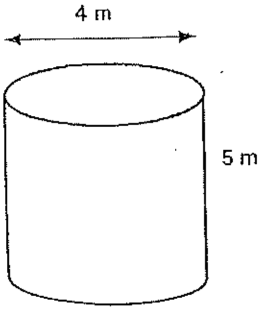
Exercise 3.9 Applications of Volume



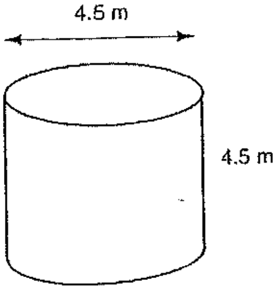
1. A cylindrical tin of diameter 2cm and height 2cm fits tightly inside a cube with measurements of 2cm.
   a. Find the volume of the cylinder and the volume of the cube.
   b. Write the volume of the cylinder as a percentage of the volume of the cube.

2. Which of these cylinders has the greatest volume?

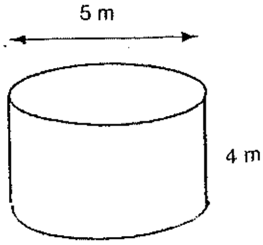
a.



b.

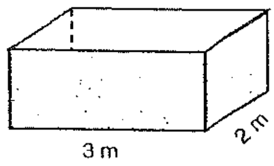


c.



3. A pipe is 4cm in diameter. If water passes through at 5cm per second, what volume of water would pass through:
   a. in a minute?
   b. in an hour?

4. A tank is filled with 12000L (12 kilolitres) of water. What is the height of the tank?



5. A 5 litre bucket is used to fill a rectangular tank of dimensions 2m x 3m x 1m. How many trips would be needed to completely fill the tank?

6. Six machines in a factory each use 460 litres of water per hour. If the machines work for 8 hours per day for 5 days per week, how much water is used in a week? (Give your answer in kilolitres)

7. A concrete pipe has an external diameter of 50cm and an internal diameter of 40cm. Concrete weighs 2 tonne per cubic metre. What would be the weight of a 6 metre length of pipe.

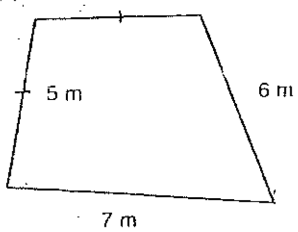
8. A solid prism has a height of 64cm and a volume of $193600cm^3$.
   a. Calculate the surface area of the top.
   b. If the top is square, what is its side length?

9. Mike's pet frog can jump to a height of 45cm. Mike has put it into a box with a volume of $0.28m^3$ and base measurements of 80cm x 70cm. Can the frog jump out of the box?

10. Mai used $2000cm^3$ of paint on a circular area with radius 3.4m. What is the thickness of the layer of paint?

Exercise 3.10

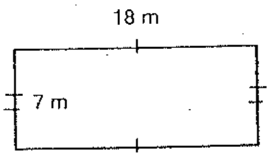
1. Find the perimeter of:



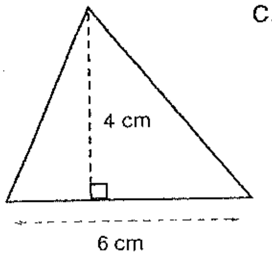
2. Calculate the circumference of a circle of radius 20cm.

3. Find the area of the following:

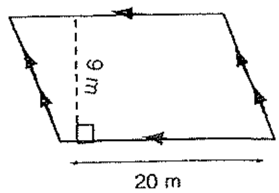
a.



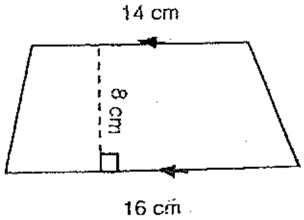
b.



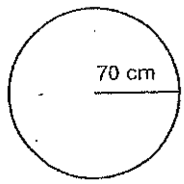
c.



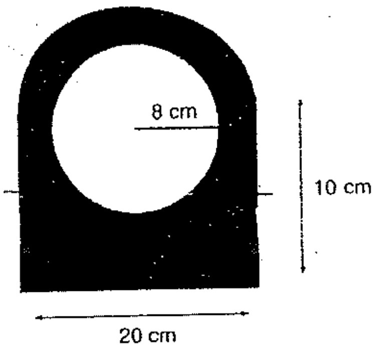
d.



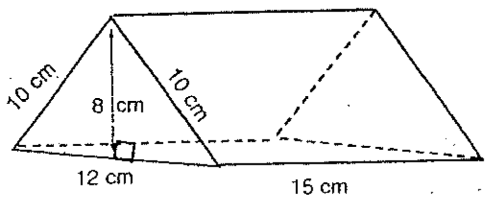
e.



4. Calculate the shaded area:



5. Find the surface area:

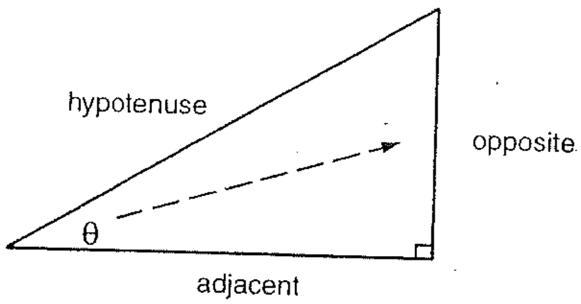


Topic Five: Trigonometry

The word "trigonometry" comes from two Greek words, **trigon** meaning a triangle and **metron** meaning a measure.

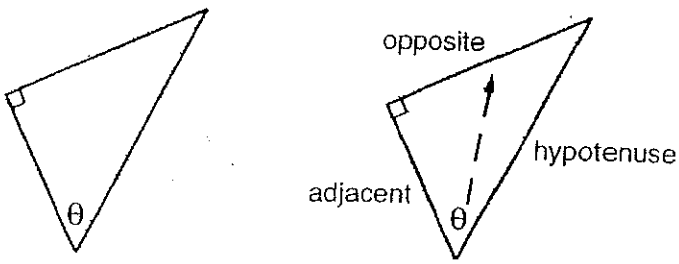
We use trigonometry to find sides and angles in right–angled triangles. Before we go on to the formulas that we will use, we need to name the sides of the triangle.

> The angle is often named with the symbol θ.
>
> The longest side is the **hypotenuse** which is opposite the right angle.
>
> The **opposite** side is opposite the angle θ.
>
> The **adjacent** side is next to θ.

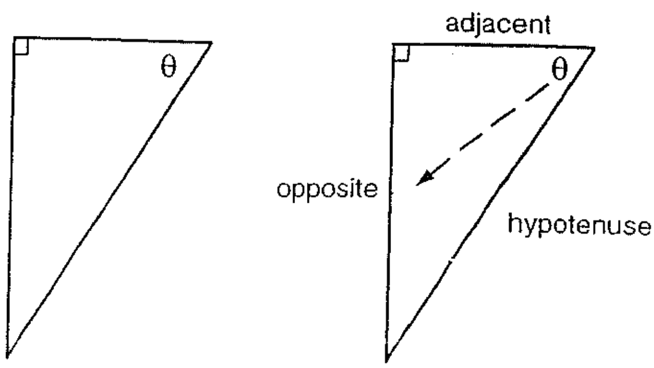


Example 1 Name the sides in the following triangles:

(a)



(b)



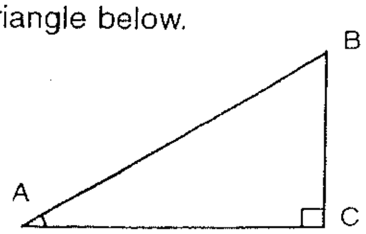
The three formulas (or ratios) we will use, sin θ, cosine θ and tangent θ (abbreviated to sin θ, cos θ, tan θ) are:

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	or	$\sin\theta = \dfrac{O}{H}$
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	or	$\cos\theta = \dfrac{A}{H}$
$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	or	$\tan\theta = \dfrac{O}{A}$

SOH Sine = opposite / Hypotenuse
CAH Cosine = Adjacent / hypotenuse
TOA Tangent = opposite / Adjacent

You need to learn these formulas. Some people use the abbreviation SOH CAH TOA to help them remember them.

Example 2 Write the formulas for sin A, cos A and tan A for the triangle below.



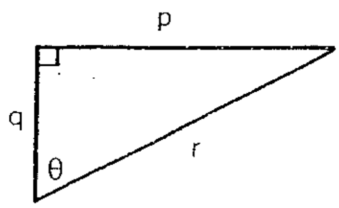
The hypotenuse is AB.
The opposite is BC.
The adjacent is AC.

So $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{BC}{AB}$

$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{AC}{AB}$

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{BC}{AC}$

Example 3 Write the ratios for the triangle below:

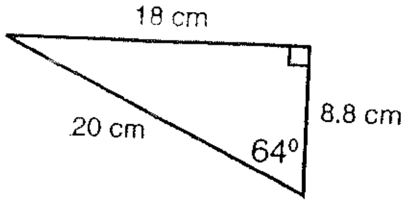


$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{p}{r}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{q}{r}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{p}{q}$

Example 4 For the triangle below, calculate the trig ratios of 64^{0} correct to 2 decimal places.

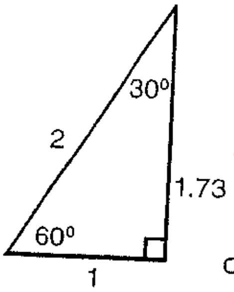


$$\sin 64^{0} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{18}{20} = 0.90$$

$$\cos 64^{0} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8.8}{20} = 0.44$$

$$\tan 64^{0} = \frac{\text{opposite}}{\text{adjacent}} = \frac{18}{8.8} = 2.25$$

Example 5 Using the triangle below, calculate sin 60^{0}, tan 60^{0} and cos 30^{0}.



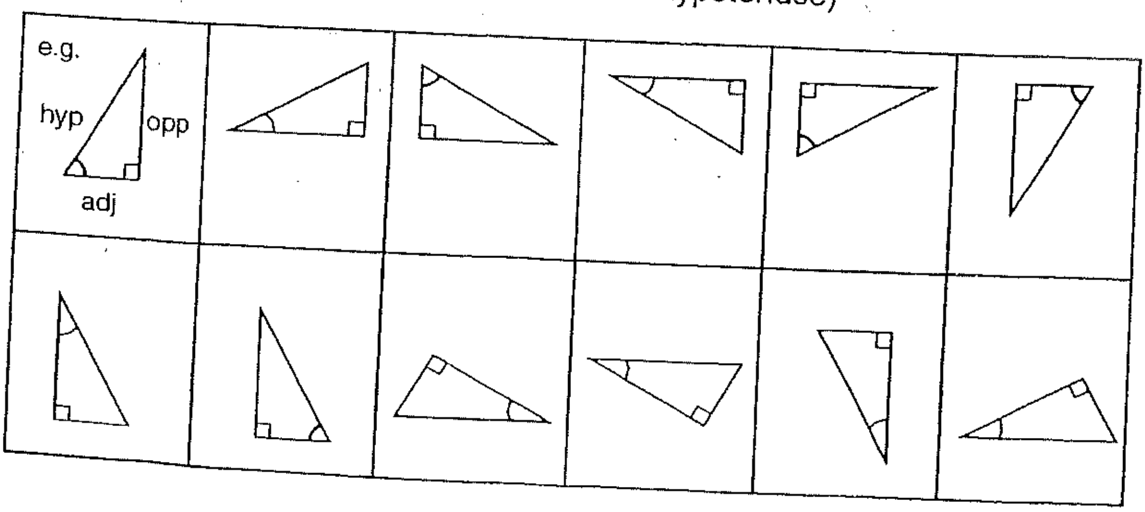
$$\sin 60^{0} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1.73}{2} = 0.865$$

$$\tan 60^{0} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1.73}{1} = 1.73$$

$$\cos 30^{0} = \frac{\text{opposite}}{\text{adjacent}} = \frac{1.73}{2} = 0.865$$

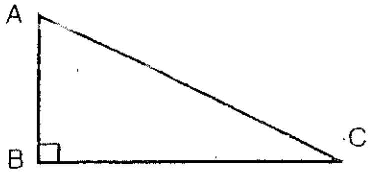
Exercise 5.1

1. Draw the following triangles in your workbook and then label each side with its name (opposite, adjacent and hypotenuse)



2. For each of the following triangles, complete the trig. ratios asked for:

(a)

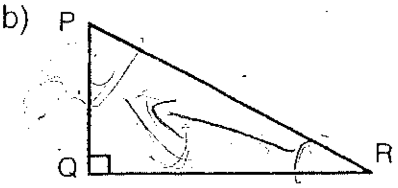


$\sin C = \frac{AB}{?}$

$\cos C = \frac{?}{?}$

$\tan C = \frac{?}{?}$

(b)

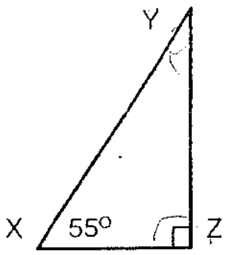


$\sin R =$ .... $\sin P =$..........

$\cos R =$ .... $\cos P =$..........

$\tan R =$ .... $\tan P =$..........

(c)

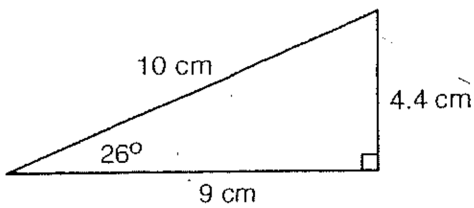


$\sin 55^\circ = \frac{YZ}{XY}$ $\sin 35^\circ =$ —

$\cos 55^\circ =$ — $\cos 35^\circ =$ —

$\tan 55^\circ =$ — $\tan 35^\circ =$ —

3. In the following triangle, complete the calculations of the values of the trig. ratios.

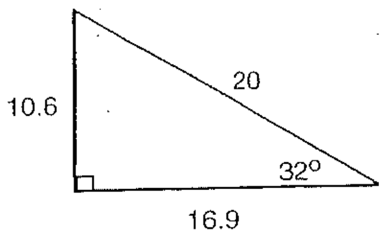


$\sin 26^\circ = \frac{\quad}{10} =$

$\cos 26^\circ =$ — $=$

$\tan 26^\circ =$ — $=$

4. Calculate the following ratios:



$\sin 32^\circ = \frac{\quad}{20} =$

$\cos 32^\circ =$ — $=$

$\tan 32^\circ =$ — $=$

$\sin 58^\circ =$ — $=$

$\cos 58^\circ =$ — $=$

$\tan 58^\circ =$ — $=$

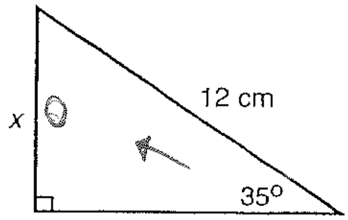
SINE
Finding the Opposite

If we are given an angle and the hypotenuse of a right-angled triangle, we can then calculate the length of the opposite side.

Examples: Find the opposite side correct to 2 decimal places.

Remember: $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

(a)

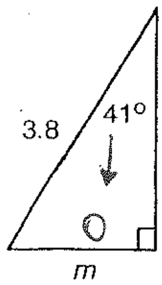


angle = 35°
hypotenuse = 12 cm
opposite = x

Steps

1. Write out the formula. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
2. Substitute the values from the triangle into the fomula. $\sin 35 = \frac{x}{12}$
3. Use a calculator or the table to find sin 35. $0.5736 = \frac{x}{12}$
4. Multiply each side by 12 to solve for x. $0.5736^{\times 12} = \frac{x}{12}^{\times 12}$
   6.88 cm = x

(b)



angle = 41°
hypotenuse = 3.8
opposite = m

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin 41 = \frac{m}{3.8}$

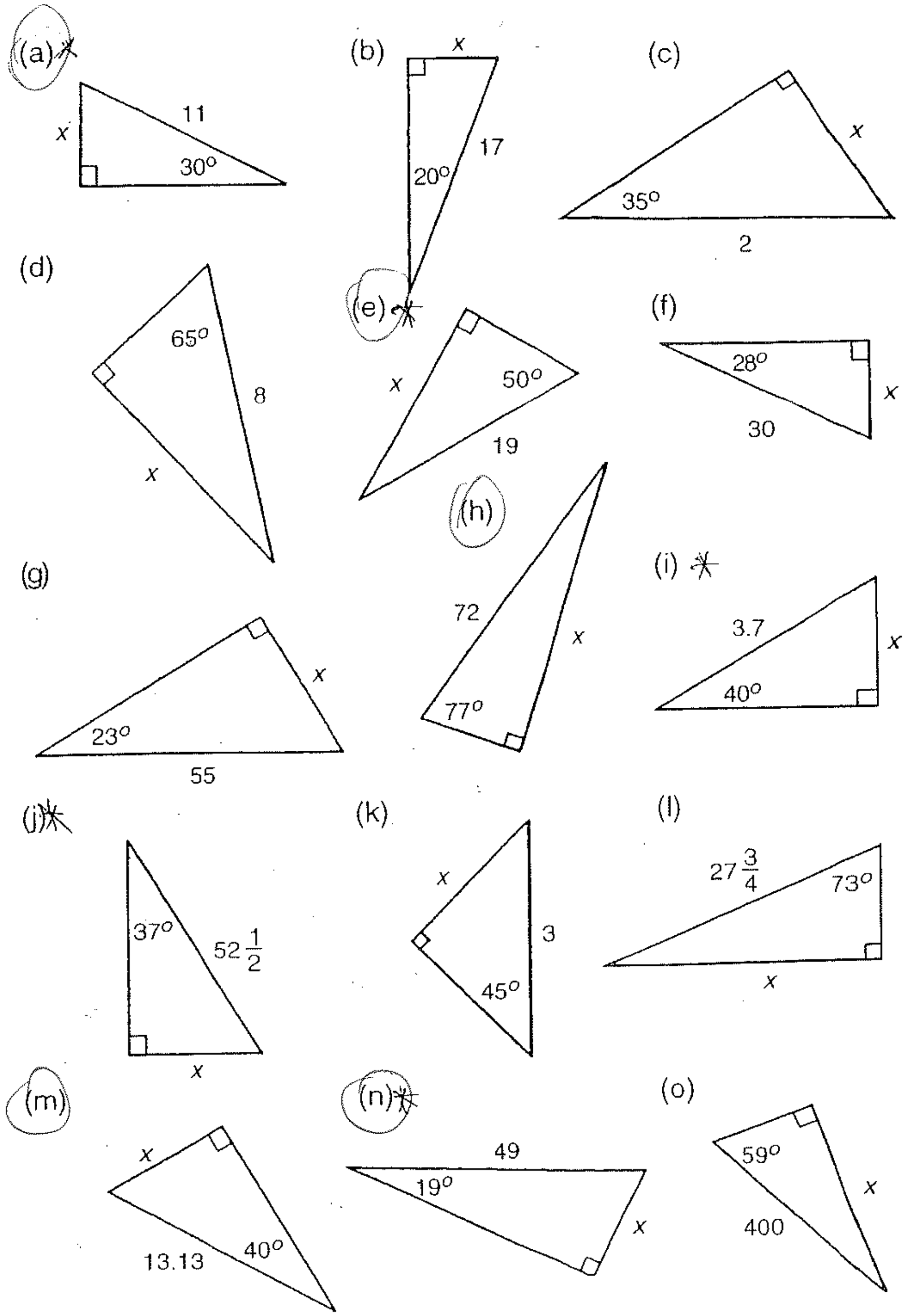
(Using calc. or tables) $0.6561 = \frac{m}{3.8}$

$0.6561^{\times 3.8} = \frac{m}{3.8}^{\times 3.8}$

2.49 = m

Exercise 5.3

1. Find the values of x in the following triangle. Give your answers to two decimal places.



2. A triangle EFG is right angled at F, side DE = 19 cm and $\angle FDE = 46^\circ$.
   (a) Draw a diagram of the triangle
   (b) Find length FE.

3. In triangle PQR, PQ = 20 m, $\angle Q = 36^\circ$, $\angle R = 90^\circ$. Find QR.

4. Triangle ABC has $\angle B = 90^\circ$, $\angle A = 73^\circ$ and AC = 20 cm.
   (a) Draw a diagram
   (b) Find the length CB
   (c) Using Pythagoras's Theorem, find the length AB.

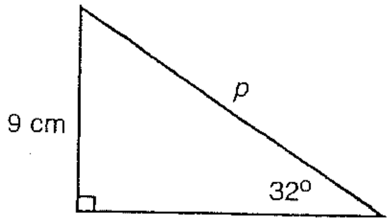
Finding the Hypotenuse

Finding the hypotenuse is more difficult because the unknown side is on the bottom (denominator) of the fraction.

It is **very** important that you show **all** working out.

Examples Find the value of p to 2 decimal places.

(a)

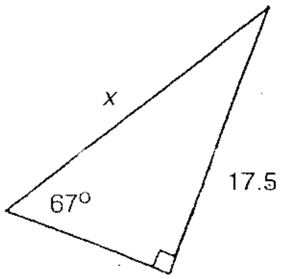


Angle = 32°
Hypotenuse = p
Opposite = 9cm

Steps

1. Write out the formula. $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
2. Substitute the values from the triangle into the formula. $\sin 32 = \dfrac{9}{p}$
3. Use a calculator or the table to find sin 32. $0.5299 = \dfrac{9}{p}$
4. Multiply both sides by p. $0.5299 \times p = 9$
5. Divide both sides by 0.5299. $p = \dfrac{9}{0.5299}$
6. Calculate p by dividing. $p = 16.98$ cm

(b)



angle = 67°
hypotenuse = x
opposite = 17.5

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 67 = \frac{17.5}{x}$$

$$0.9205 = \frac{17.5}{x}$$

$$0.9205 \times x = 17.5$$

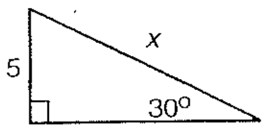
$$x = \frac{17.5}{0.9205}$$

$$= 19.01$$

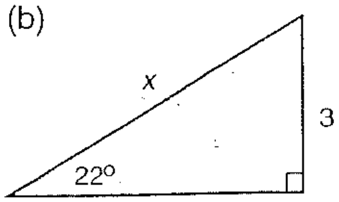
Exercise 5.4

1. Find the values of x in the following triangles. Give your answer to 2 decimal places.

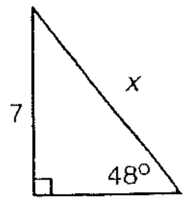
(a)



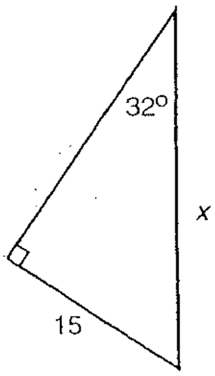
(b)



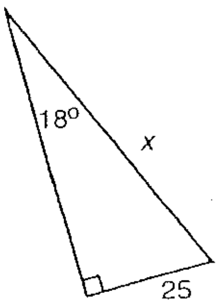
(c)



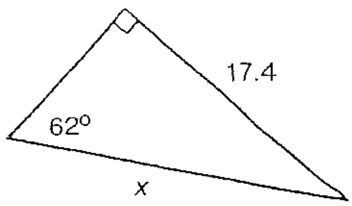
(d)



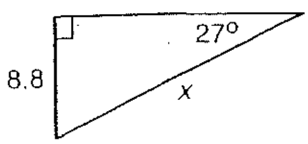
(e)



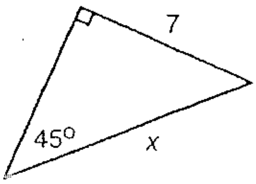
(f)



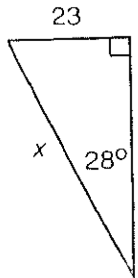
(g)



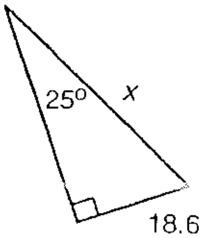
(h)



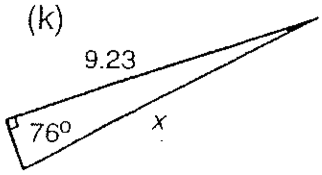
(i)



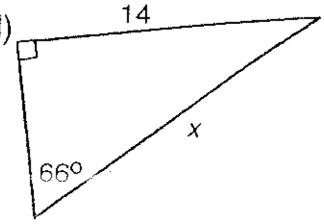
(j)



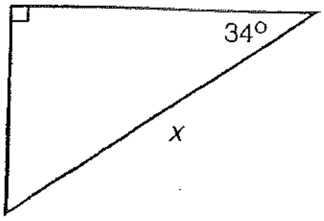
(k)



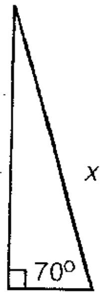
(l)



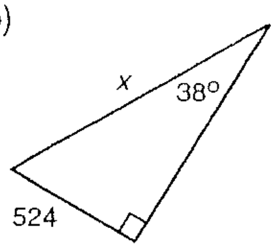
(m)



(n)



(o)



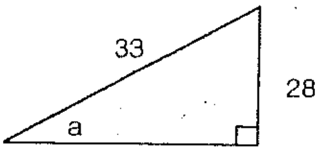
2. In triangle ABC, $\angle B = 90^\circ$, $\angle C = 27^\circ$ and side AB = 13 m.
   (a) Draw a diagram
   (b) Find the length AC.

3. Triangle DEF is right angled at D, side DF = 21.8 cm and $\angle DEF = 48^\circ$.
   (a) Draw a diagram
   (b) Find the length EF.

4. In triangle XYZ, $\angle Z = 90^\circ$, $\angle X = 84^\circ$ and side ZY = 100 mm.
   (a) Draw a diagram
   (b) Find the length XY
   (c) Use Pythagoras's Theorem to calculate the length XZ.

Finding the Angle

Example 1



angle = a
hypotenuse = 33
opposite = 28

On your Calculator press [Shift] [Sin] $\frac{28}{33}$

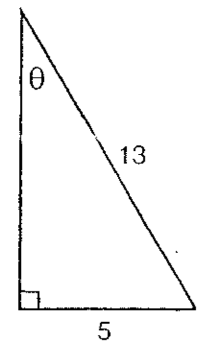
or

[Shift] [Sin] 0.8485

Steps

1. Write out the formula. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
2. Substitute the values from the triangle into the formula. $\sin a = \frac{28}{33}$
3. Divide the fraction. $\sin a = 0.8485$
4. Use inverse (or 2nd function) sin key on the calculator or work backwards from the table. $a = 58.05$
   $= 58^\circ$

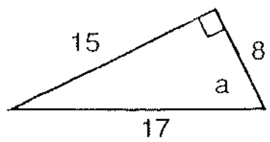
Example 2



$$\text{angle} = \theta$$
$$\text{hypotenuse} = 13$$
$$\text{opposite} = 5$$
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin\theta = \frac{5}{13}$$
$$= 0.3846$$
$$\theta = 22.62^{\circ}$$

(23° to the nearest whole degree)

Example 3



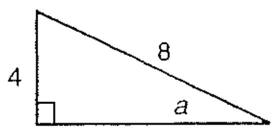
$$\text{angle} = a$$
$$\text{hypotenuse} = 17$$
$$\text{opposite} = 15 \quad \text{(8 is adjacent)}$$
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin a = \frac{15}{7}$$
$$= 0.8824$$
$$a = 61.93$$

(62° to the nearest whole degree)

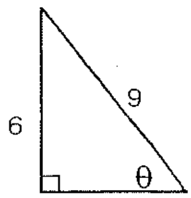
Exercise 5.5

1. Find the angle in the following triangles.

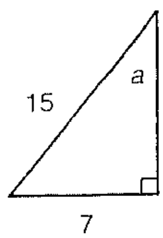
(a)



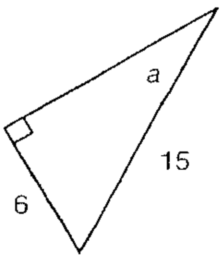
(b)



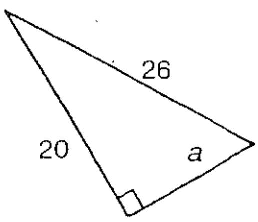
(c)



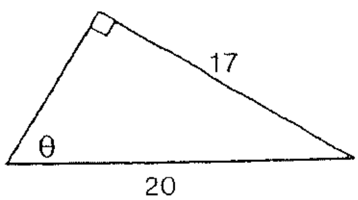
(d)



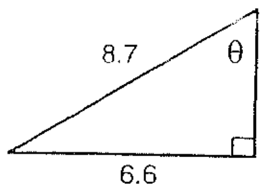
(e)



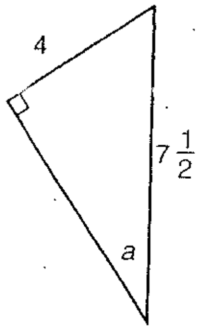
(f)



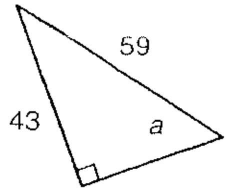
(g)



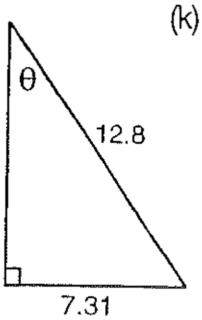
(h)



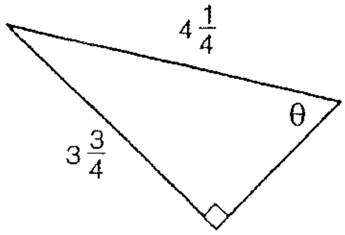
(i)



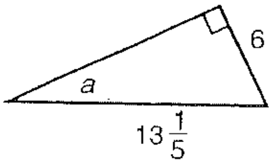
(j)



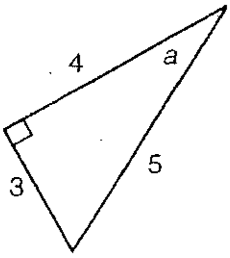
(k)



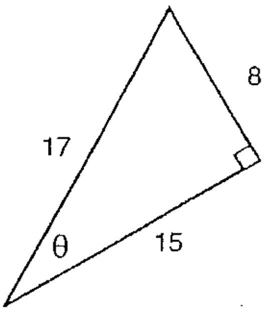
(l)



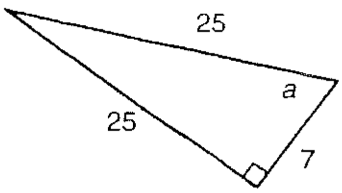
(m)



(n)



(o)



2. In triangle STV, $\angle T = 90^{\circ}$, length ST = 6 cm and SV = 9.5 cm.
   (a) Draw a diagram
   (b) Find angle $\angle V$
   (c) Use Pythagoras's Theorem to find length TV
   (d) Find angle $\angle S$

3. In triangle ABC, $\angle ACB = 90^{\circ}$, CB = 11 and AB = 12.
   (a) Find $\angle$ CAB
   (b) Find AC
   (c) Find $\angle$ ABC

COSINE

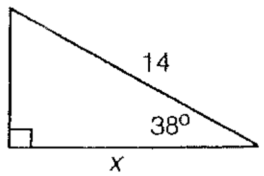
Remember $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

The methods we will use to solve triangles using cosine are the same as we used for sine.

Finding the Adjacent

Examples Find the adjacent side giving answers to 2 decimal places.

(a)



angle = 38°

hypotenuse = 14

adjacent = x

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

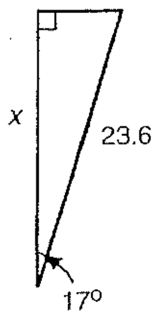
$$\cos 38 = \frac{x}{14}$$

$$0.7880 = \frac{x}{14}$$

$$0.7880 \times 14 = \frac{x}{14} \times 14$$

$$11.03 = x$$

(b)



angle = 17°

hypotause = 23.6

adjacent = x

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 17 = \frac{x}{23.6}$$

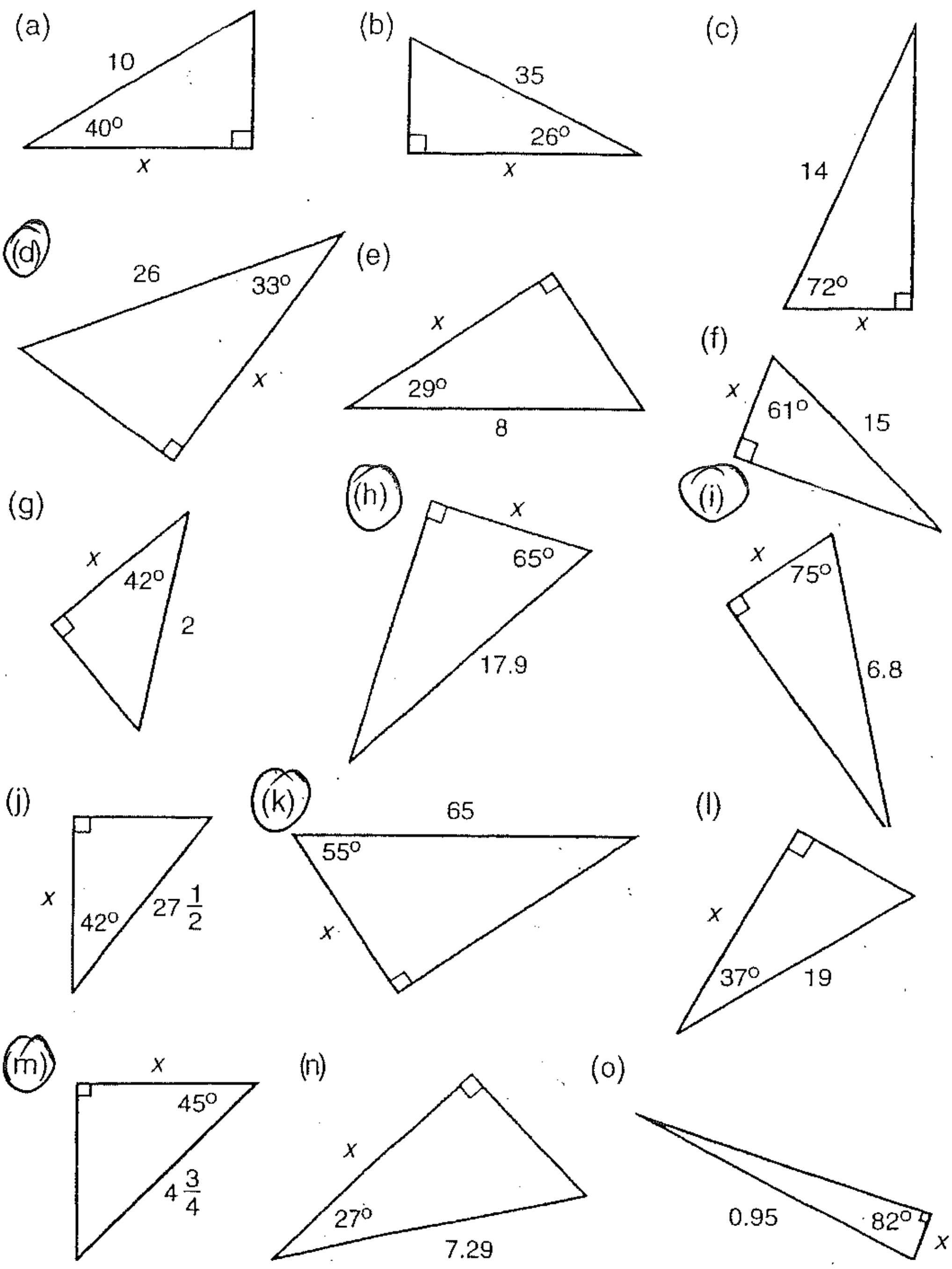
$$0.9563 = \frac{x}{23.6}$$

$$0.9563 \times 23.6 = x$$

$$22.57 = x$$

Exercise 5.6

1. Find the values of x in the following triangles. Give your answers to 2 decimal places.

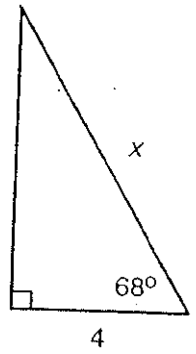


2. In triangle MNP, $\angle P = 90^\circ$, $\angle M = 86^\circ$ and NM = 14 cm.
   (a) Find length PM
   (b) Use Pythgoras's Theorem to find length NP.

Finding the Hypotenuse

Examples Find the value of the hypotenuse

(a)



angle = 68°
hypotenuse = x
adjacent = 4

$$\cos\ \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 68 = \frac{4}{x}$$

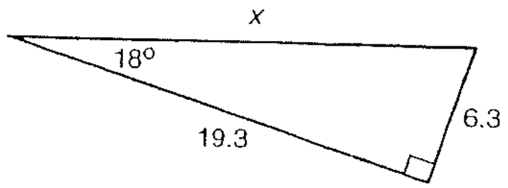
$$0.3746 = \frac{4}{x}$$

$$0.3746 \times x = 4$$

$$x = \frac{4}{0.3746}$$

$$= 10.68$$

(b)



angle = 18°
hypotenuse = x
adjacent = 19.3

$$\cos\ \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 18 = \frac{19.3}{x}$$

$$0.9511 = \frac{19.3}{x}$$

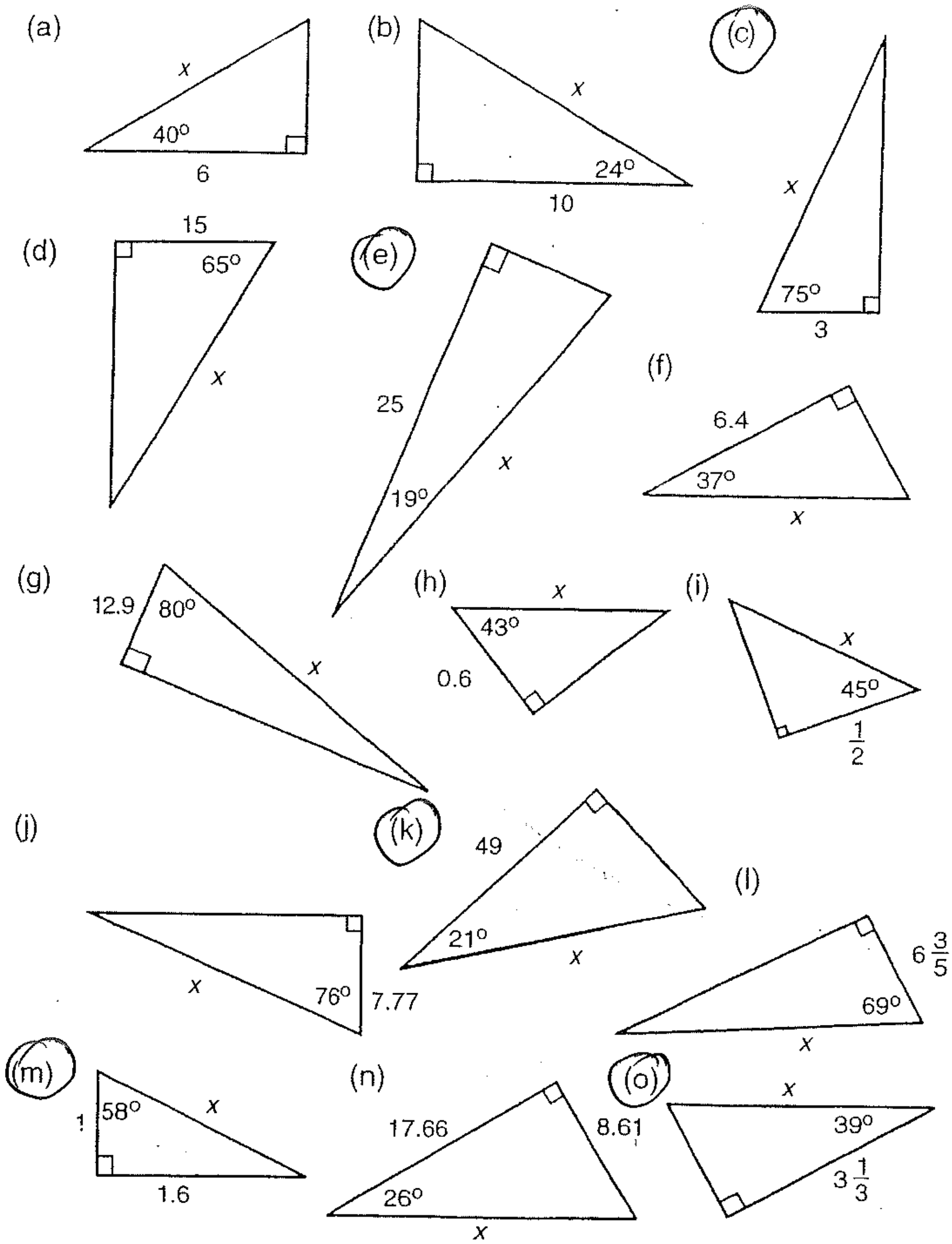
$$0.9511 \times x = 19.3$$

$$x = \frac{19.3}{0.9511}$$

$$= 20.29$$

Exercise 5.7

1. Find the values of x in the following triangles. Give your answer to 2 decimal places.



* 2. In triangle ABC, $\angle A = 90^{\circ}$, $\angle B = 33^{\circ}$ and length AB = 452 cm.

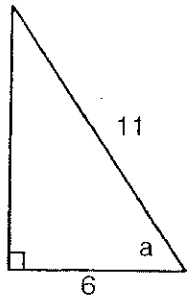
(a) Find length CB

(b) Use Pythagoras's Theorem to find length CA.

Finding the Angle

Examples Find the angle in the following triangles:

(a)



angle = a
hypotenuse = 11
adjacent = 6

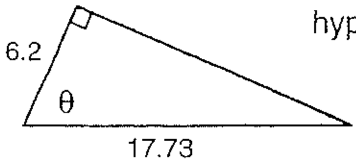
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos a = \frac{6}{11}$$

$= 0.5455$

$a = 56.94^{\circ}$ (or 57°)

(b)



angle = θ
hypotenuse = 17.73
adjacent = 6.2

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{6.2}{17.73}$$

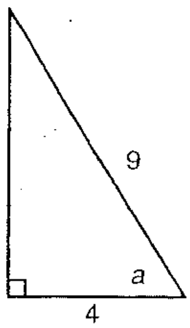
$= 0.3497$

$\theta = 69.53^{\circ}$ (or 70°)

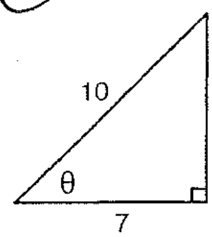
Exercise 5.8

1. Find the angle in the following triangles:

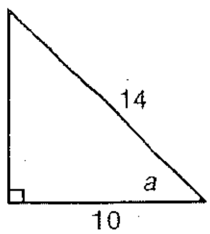
(a)



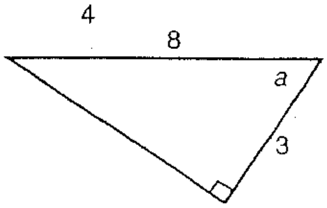
(b)



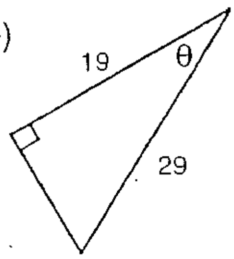
(c)



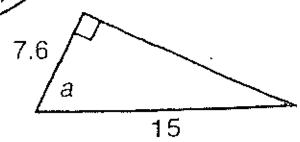
(d)



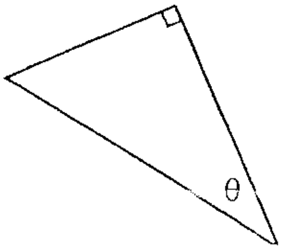
(e)



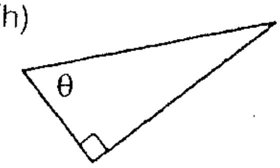
(f)



(g)

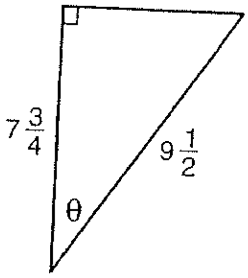


(h)



(i)

(j)



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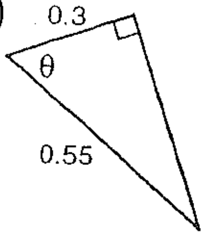
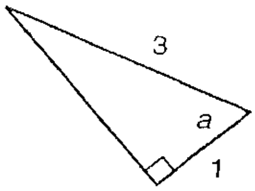
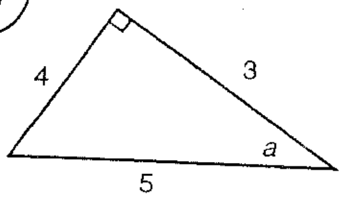


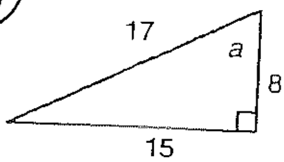
(l)



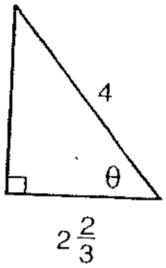
(m)



(n)



(o)



✱ 2. In triangle LMN, $\angle L = 90^{\circ}$, length LM = 16 and length LN = 12.

(a) Use Pythagoras's Theorem to find length MN

(b) Find $\angle N$

(c) Find $\angle M$.

TANGENT

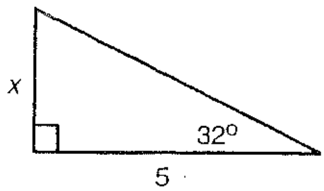
> Remember: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

Tangent questions are easy to recgonise since the triangle will not have any value written on the hypotenuse.

Finding the Opposite

Examples Find the value of the opposite side to 2 decimal places.

(a)



angle = 32°
opposite = x
adjacent = 5

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

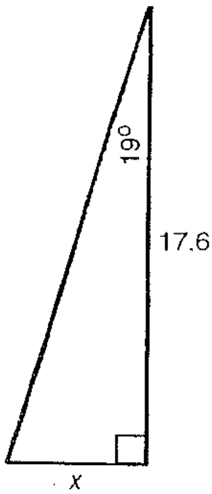
$$\tan 32 = \frac{x}{5}$$

$$0.6249 = \frac{x}{5}$$

$$0.6249 \times 5 = x$$

$$3.12 = x$$

(b)



angle = 19°
opposite = x
adjacent = 17.6

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 19 = \frac{x}{17.6}$$

$$0.3443 = \frac{x}{17.6}$$

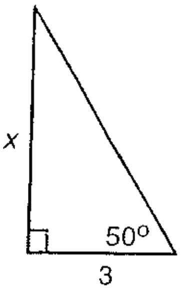
$$0.3443 \times 17.6 = x$$

$$6.06 = x$$

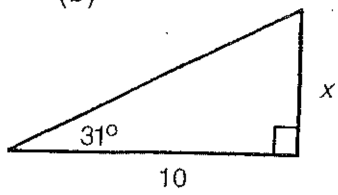
Exercise 5.9

1. Find the value of x, to 2 decimal places.

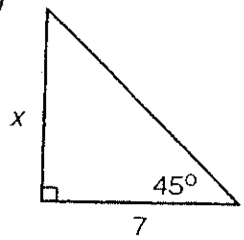
(a)



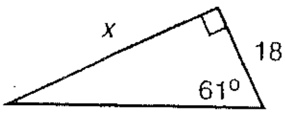
(b)



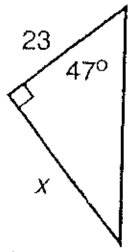
(c)



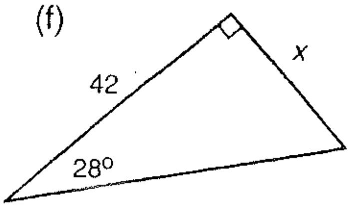
(d)



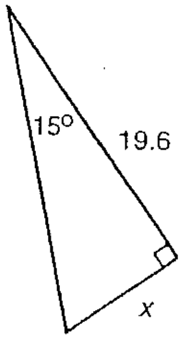
(e)



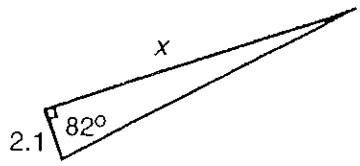
(f)



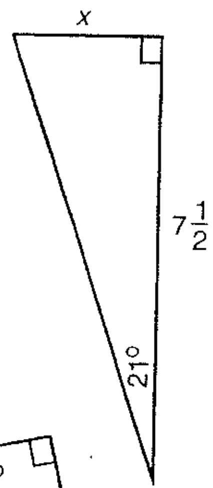
(g)



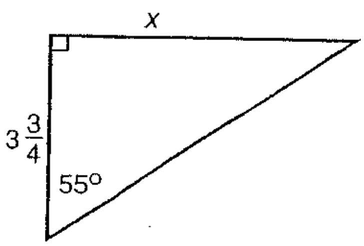
(h)



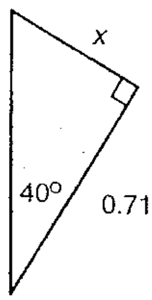
(i)



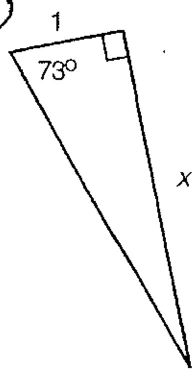
(j)



(k)



(l)

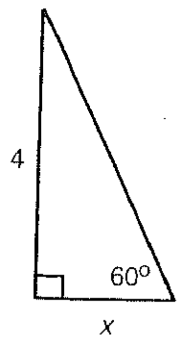


2. In triangle XYZ, $\angle Y = 90^\circ$, $\angle X = 63^\circ$ and length XY = 11.21 cm.
   (a) Draw a diagram
   (b) Find length ZY
   (c) Use Phythagoras's Theorem to find XZ
   (d) Find $\angle Z$.

Finding the Adjacent

Examples Find the length of the adjacent, giving answers to 2 decimal places.

(a)



angle = 60°

opposite = 4

adjacent = x

$$\tan\ \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 60 = \frac{4}{x}$$

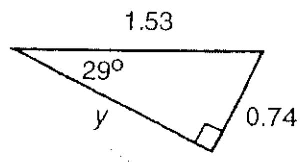
$$1.7321 = \frac{4}{x}$$

$$1.7321 \times x = 4$$

$$x = \frac{4}{1.7321}$$

$$= 2.31$$

(b)



angle = 29°

opposite = 0.74

adjacent = y

$$\tan\ \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 29 = \frac{0.74}{y}$$

$$0.5543 = \frac{0.74}{y}$$

$$0.5543 \times y = 0.74$$

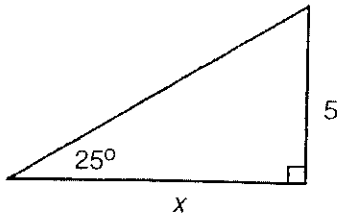
$$y = \frac{0.74}{0.5543}$$

$$= 1.34$$

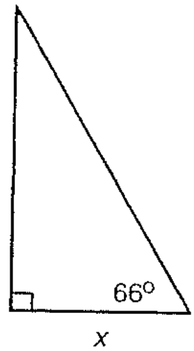
Exercises 5.10

1. Find the value of x to 2 decimal places.

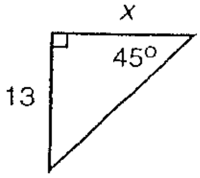
(a)



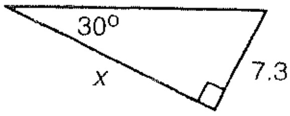
(b)



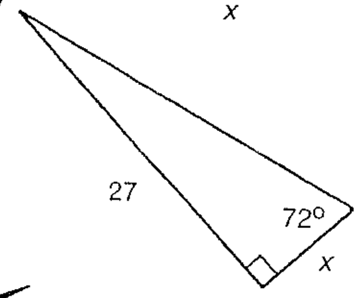
(c)



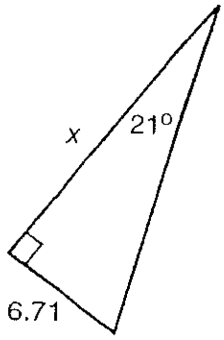
(d)



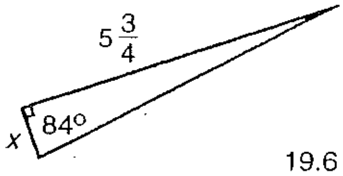
(e)



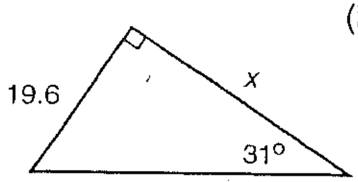
(f)



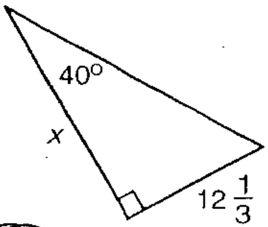
(g)



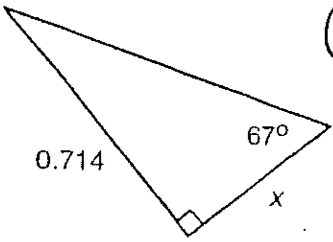
(h)



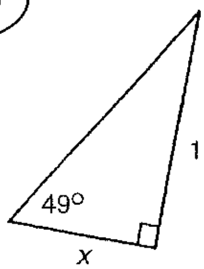
(i)



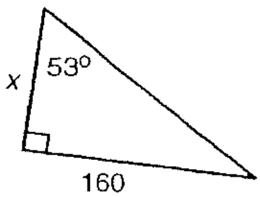
(j)



(k)



(l)

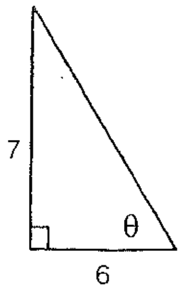


2. In the triangle HKL, $\angle HKL = 90^\circ$, $\angle KHL = 23^\circ$ and length KL = 47 cm.
   - (a) Draw a diagram
   - (b) Find length HK
   - (c) Find length HL
   - (d) Find $\angle HLK$.

Finding the Angle

Examples: Find the value of the angle.

(a)



angle $= \theta$

opposite $= 7$

adjacent $= 6$

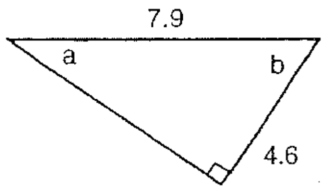
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Shift tan $\frac{7}{6}$ ← $\tan\theta = \frac{7}{6}$

$= 1.1667$

$\theta = 49.4^{\circ}$ (or 49°)

(b)



angle = a

opposite = 4.6

adjacent = 7.9

Finding a .... $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$$\tan a = \frac{4.6}{7.9}$$

$= 0.5823$

$a = 30.21^{\circ}$ (or 30°)

Finding b ....

Remember the angles inside a triangle add up to 180°.

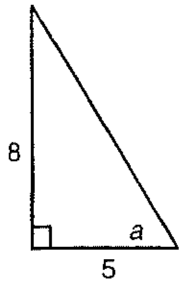
$b = 180 - 30.21 - 90^{\circ}$

$= 59.79$ (or 60°)

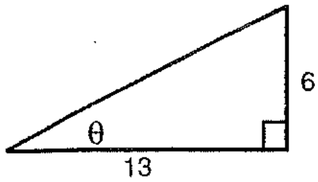
Exercise 5.11

1. Find the value of the angle.

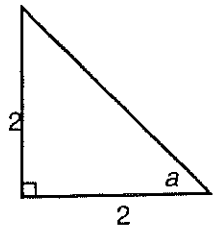
(a)



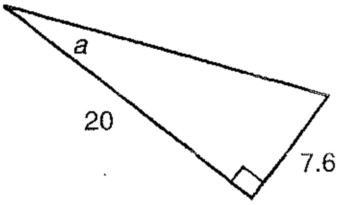
(b)



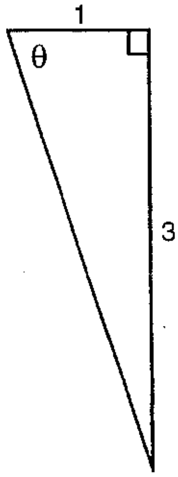
(c)



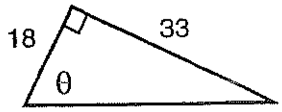
(d)



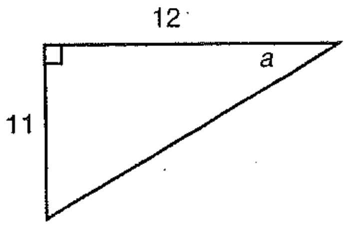
(e)



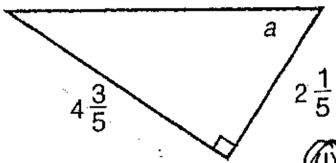
(f)



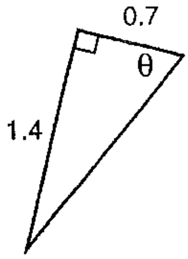
(g)



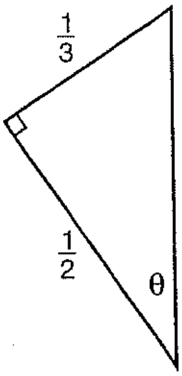
(h)



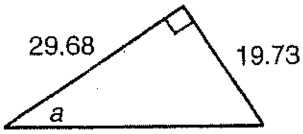
(i)



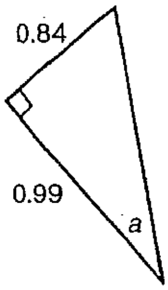
(j)



(k)



(l)



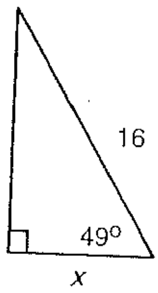
2. In triangle VWY, $\angle V = 90^\circ$, VW = 72 cm and VY = 104 cm.
   (a) Find $\angle$ W
   (b) Find $\angle$ Y
   (c) Find length WY.

Using Sine, Cosine and Tangent

To find a side or angle in a right- angled triangle you need to first decide whether to use the formula for sine, cosine or tangent.

Example 1 Find the value of the pronumeral in the following:

(a)



Here we have ...

angle = 49°

hypotenuse = 16

adjacent = x

So we need to use **cosine**.

$$\cos\ \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

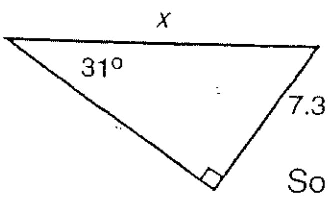
$$\cos 49 = \frac{x}{16}$$

$$0.6561 = \frac{x}{16}$$

$$0.6561 \times 16 = x$$

$$10.5 = x$$

(b)



Here we have ...

angle = 31°

hypotenuse = x

opposite = 7.3

So we need to use **sine**.

$$\sin\ \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

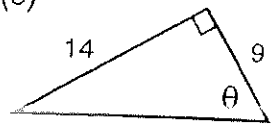
$$\sin 31 = \frac{7.3}{x}$$

$$0.5150 \times x = \frac{7.3}{x}$$

$$x = \frac{7.3}{0.5150}$$

$$= 14.17$$

(c)



Here we have ...

angle = θ

opposite = 14

adjacent = 9

So we need to use **tangent**.

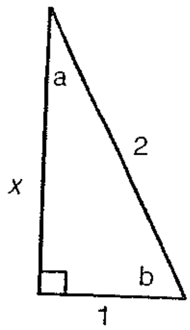
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{14}{9}$$

$$= 1.5556$$

$$\theta = 57.27^\circ \text{ (or } 57^\circ)$$

Example 2 Find all pronumerals.



To find a ...

angle = a

opposite = 1

hypotenuse = 2

therefore **sine**

$$\sin a = \frac{1}{2}$$

$$= 0.5$$

$$a = 30^\circ$$

$$b = 180^\circ - 90^\circ - 30^\circ$$
$$= 60^\circ$$

To find *x* you could use cosine, tangent or Pythagoras's Theorem.

Using cosine ... .

angle = 30°

adjacent = x

hypotenuse = 2

$$\cos 30 = \frac{x}{2}$$

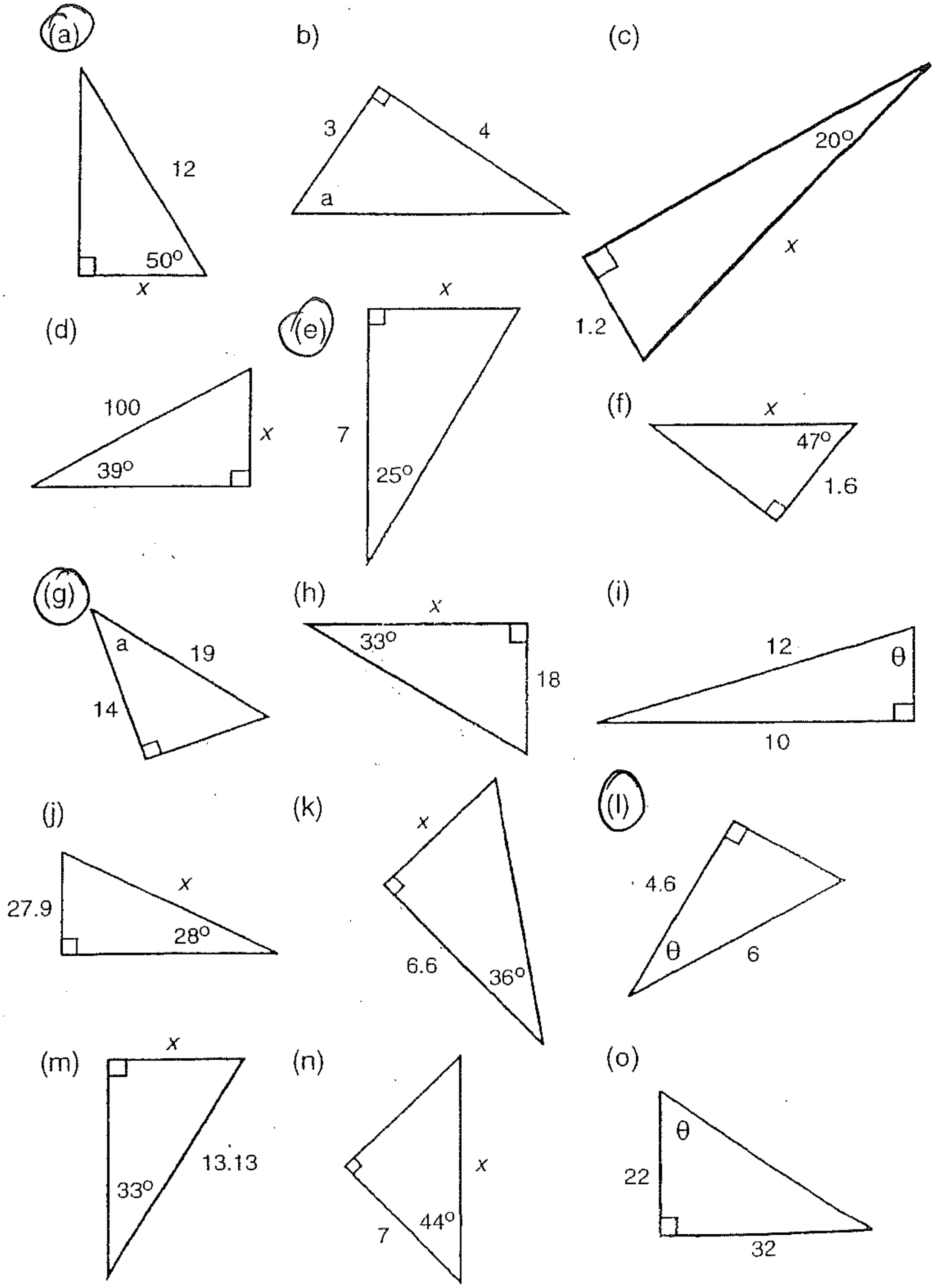
$$0.8660 = \frac{x}{2}$$

$$0.8660 \times 2 = x$$

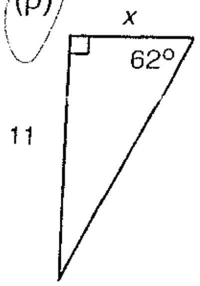
$$1.732 = x$$

Exercise 5.12

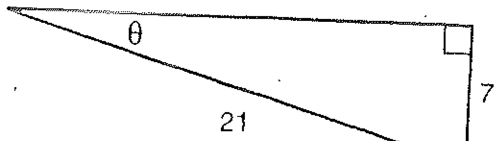
1. For the following triangles, first decide whether to use **sine**, **cosine** or **tangent** and then find the value of the pronumeral.



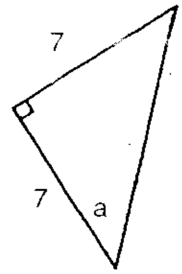
(p)



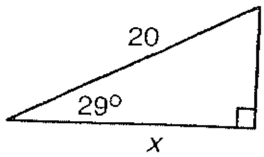
(q)



(r)



(s)



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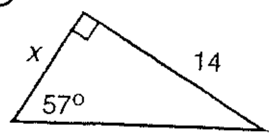

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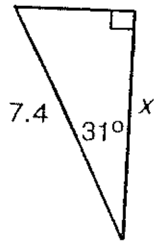

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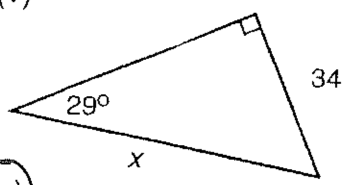

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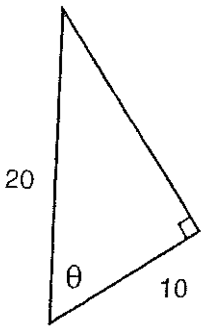

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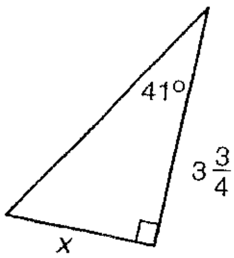

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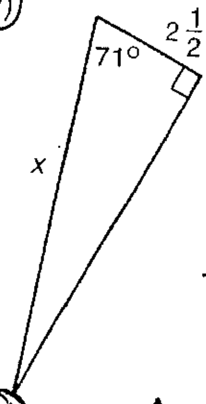

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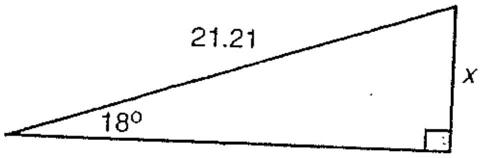

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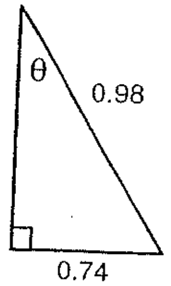

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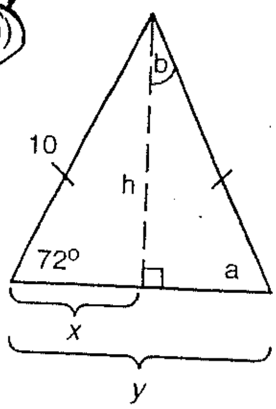

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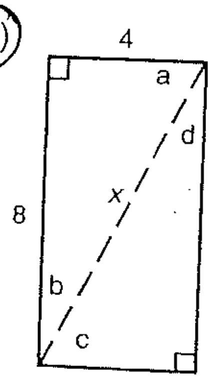
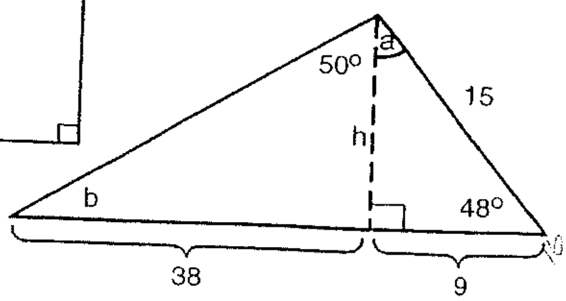
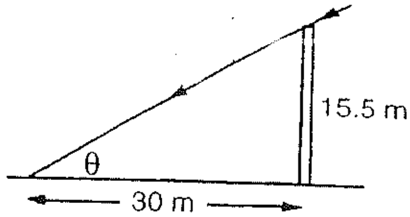


(ad)

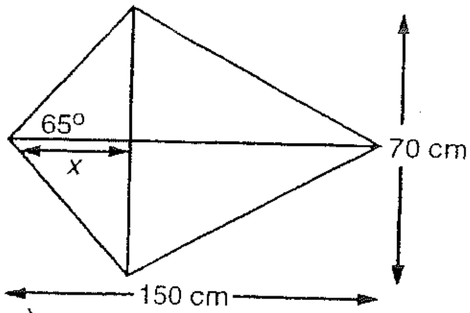


3. A vertical pole 15.5 m high casts a shadow 30 m long on the horizontal ground. What is the angle of elevation of the sun? (Give answer correct to the nearest degree.)



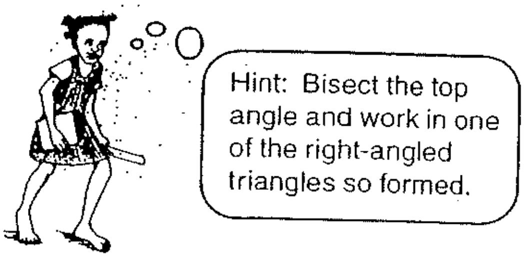
4. A ladder 3750 mm long leans against a vertical wall touching it at a height of 2900 mm.
   (a) What angle does the ladder make with the wall?
   (b) How far from the bottom of the wall is the foot of the ladder?
   (Give angles correct to the nearest degree and distance correct to the nearest millimetre.)

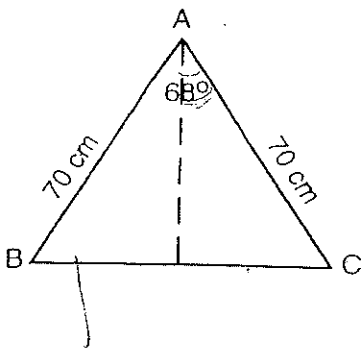
5. A kite is 150 cm long and 70 cm wide.
   The smaller edge makes and angle of 65° as shown.
   What is the length of x?



(Give answer correct to the nearest cm.)

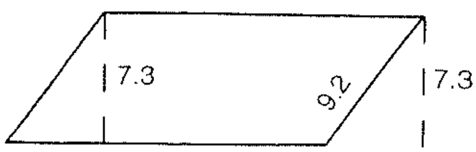
6. An isosceles triangle has equal sides of 70 mm and the angle between these sides is 68°. Find the length of the base.





(Give answer correct to the nearest mm.)

7. A parallelogram has vertical height 7.3 cm and slant height 9.2 cm. Find the sizes of the angles in the parallelogram.



(Give answers correct to the nearest degree.)

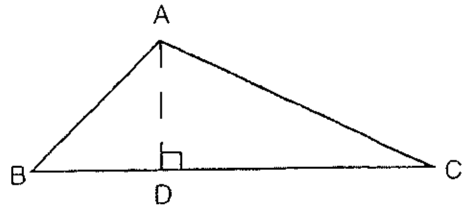
8. In triangle ABC, $\angle B = 55^{\circ}$, $\angle C = 28^{\circ}$, side AB = 20 cm.
(Give answers correct to the nearest cm.)

(a) Find the vertical height AD.

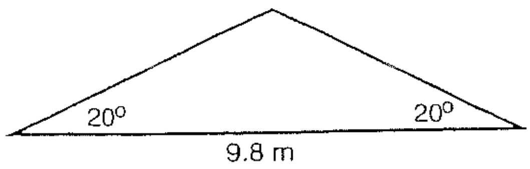
(b) Find the length BD.

(c) Find the length of side AC.
(Work in ΔADC.)

(d) Find length DC and thus the length of the side BC.



9. The diagram shows a roof with a span of 9.8 m and a pitch of 20°. Find the vertical height of the roof.



(Give answer correct to the nearest cm.)

10. A rectangular paddock is 350 m wide and 580 m long. Use Pythagoras's theorem to find the length of the diagonal (to the nearest m). Calculate $\tan\theta$ and hence find the angle it makes with the shorter sides.