“Quête pour l’excellence”
“Yumi eim blo kasem exselens”

Government of Vanuatu
PMB 9016, Port Vila, Vanuatu
Telephone: 23122 / 22323 Email admin@centralschool.edu.vu

Central School

Home School Package

Year 13: MATHEMATICS WITH CALCULUS

Ministry of Education and Training/Ministère de l'Education et de la Formation
Republic of Vanuatu/République du Vanuatu

HOME SCHOOL PACKAGE CONTENT

Contents

Lesson activity 1

Teachers

Names :
- Mr Randal Taforua
- Miss Carla Lishi

Subject : MATHEMATICS WITH CALCULUS

Date

Week 5
Monday 15th June, 2020
Tuesday 16th June, 2020

Title

Strand 3: DIFFERENTIATION
Sub strand 3.1: Differentiation Basic Skills

Lesson Topic: **Piecewise functions**

Learning outcomes

Students should be able to:

SUB-STRAND	SLO #	SPECIFIC LEARNING OUTCOME	SKILL LEVEL	SLO CODE	Achieved Yes/No
3.1	1	**identify** features of a piecewise function of f(a)	1	**Cal3.1.1.1**	
	2	**identify** features of a piecewise function if the function is discontinuous	1	**Cal3.1.1.2**	
	3	**identify** features of a piecewise function if the limit exits	1	**Cal3.1.1.3**	
	4	**identify** features of a piecewise function if the function is differentiable	1	**Cal3.1.1.4**	

Introduction

This lesson focuses on the piecewise function and its main features. A basic background on piecewise function is useful when working with derivatives of other functions.

Don't panic!

Learners notes

PIECEWISE FUNCTIONS

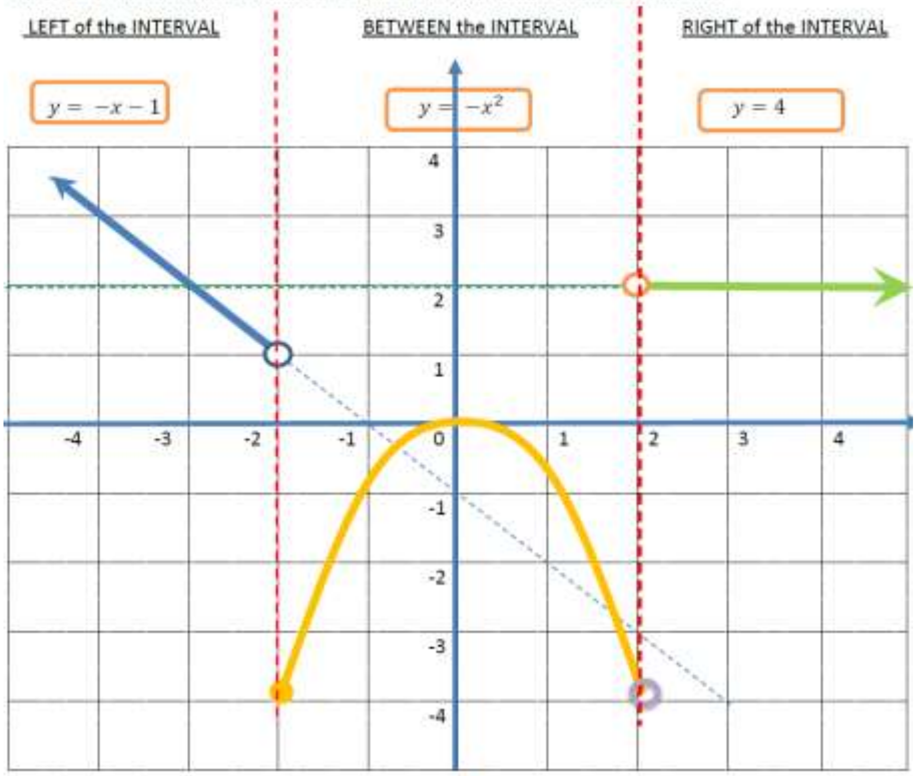
A piecewise function is a concept that is very similar to that of a "**jig-saw**" puzzle. The "*pieces*" of different functions are all drawn up on one pair of axes. This is possible by giving certain parameters to these functions, which are captured on a pair of axes, based on the contours specified on those parameters. The parameters are known as the "domain" of the "pieces" that make up the **interval** of each function. A piecewise function can accommodate *several* functions [@ Year 13 Level expect 3 or 4 functions] defined over specific intervals.

An example of a piecewise function is given below:

$$f(x) = \begin{cases} -x-1 & , \ x < -2 \\ -x^2 & , \ -2 \le x < 2 \\ 2 & , \ x \ge 2 \end{cases}$$

You will notice that there are 3 functions involved, ie.

	<u>Functions</u>		<u>Interval</u> (parameters)
	$y = -x-1$	;	$x < -2$
	$y = -x^2$	;	$-2 \le x < 2$
	$y = 2$	;	$x > 2$



FEATURES OF A PIECEWISE FUNCTION

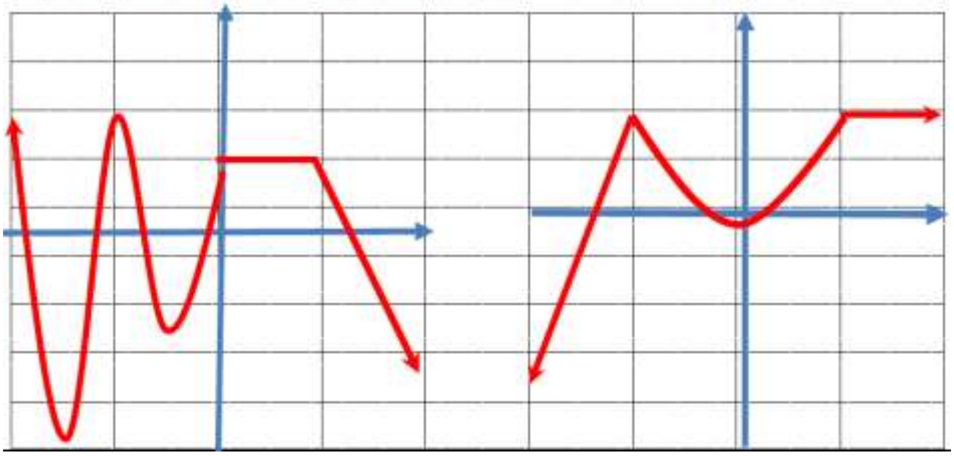
When observing the piecewise function above, there are some notable features that stand-out to make a piecewise function unique and set apart from other graphs. The following features will be discussed in detail as follows:

- Points of Continuity/Continuous
- Points of Discontinuity/Discontinuous
  - Jumps
  - Holes
- Points of Non-differentiability
- Points where the LIMIT exists
- Points where the functions are defined.

POINTS OF CONTINUITY / CONTINUOUS

The contours of the piecewise function graph can be likened to an "*electrical circuit*". As long as there is a connection in the set-up, then there will always be a flow of current or electricity. In a piecewise function, this is also evident. If the graph is fluent and continues without any interruptions, then it portrays a segment of CONTINUITY or CONTINUOUS flow.

A function is deemed continuous if its graph can be drawn without lifting pen from paper. The following functions are classified as being CONTINUOUS.



POINTS OF DISCONTINUITY/ DISCONTINUOUS

In an electrical circuit, if there is a "break" in the connection, then electricity will not flow. Likewise in a piecewise function, if there is a break in a function, then it resembles a place of DISCONTINUITY or DISCONTINUOUS flow. A break in the function can come in the form of a "**jump**" or a "**hole**" or **asymptotes.**

A JUMP

There's a HOLE at x = 1

FINDING LIMITS ON A PIECEWISE FUNCTION

A limit will exist, if the value when approaching "x" from the LEFT, and the value when approaching "x" from the RIGHT are the same.

This can be noted using the mathematical shorthand, $\boxed{\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)}$ = L

Using the FEATURES of a piecewise function, a LIMIT will exist at $x = a$ if :

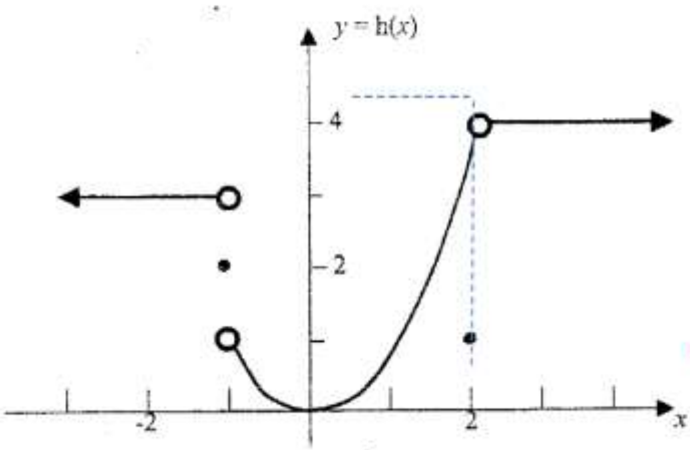
- it is **CONTINUOUS** at x = a , then $f(a) = limit$
- there is a **HOLE** at x = a , then $f(a) = limit$

A limit **<u>will not</u>** exist if the limit when approaching "x" from the left, differs from that when approaching "x" from the right. ie $\boxed{\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+}} f(x)$ A LIMIT will <u>NOT EXIST</u> if:

- there is a **JUMP** at x = a
- there is an **ASYMPTOTE** at x = a

<u>Example</u>: The piecewise function $h(x)$ is given below:

Find: i.) lim



at x = -1

ii.) lim,

$\therefore$ *corresponding y value is the LIMIT.*

$$\lim_{x\to 2} h(x) = \mathbf{4}$$

DIFFERENTIABILITY

A function will be differentiable at a point, if it is defined at that point.

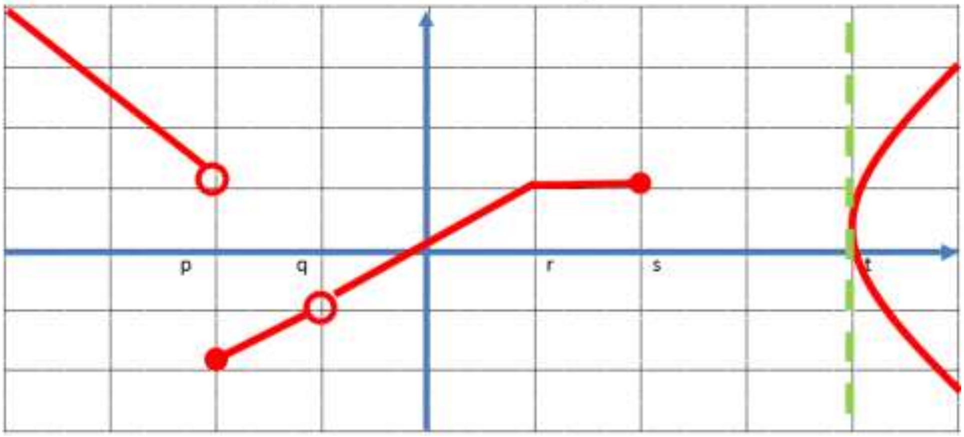
The alternative to understanding this notion, is to find where the function is <u>NOT</u> differentiable. If we are able to find the points where the functions are NOT DIFFERENTIABLE, then the points that are left out, is where the function is DIFFERENTIABLE.

POINTS OF NON-DIFFERENTIABILITY

- Jumps
- Holes
- Sharp corners / Sharp turns
- At $x = a$ there are NO VERTICAL TANGENTS

Example:

With reasons explain why $h(x)$ is NOT DIFFERENTABLE from $p - t$.



- $h(x)$ *is not differentiable at* $x = p$ *because there is* ***a JUMP*** *at* p *(discontinuous)*
- $h(x)$ *is not differentiable at* $x = q$ *because there is* ***a HOLE*** *at* q *(discontinuous)*
- $h(x)$ *is not differentiable at* $x = r$ *because it has a* ***SHARP TURN*** *or* ***CORNER***
- $h(x)$ *is not differentiable at* $x = s$ *because the* ***FUNCTION ends*** *here.*
- $h(x)$ *is not differentiable at* $x = t$ *because there is a* ***VERTICAL TANGENT*** *at* t

A function is DIFFERENTIAL if it is **CONTINUOUS**, and you're able to draw a **non-vertical tangent** at any point.

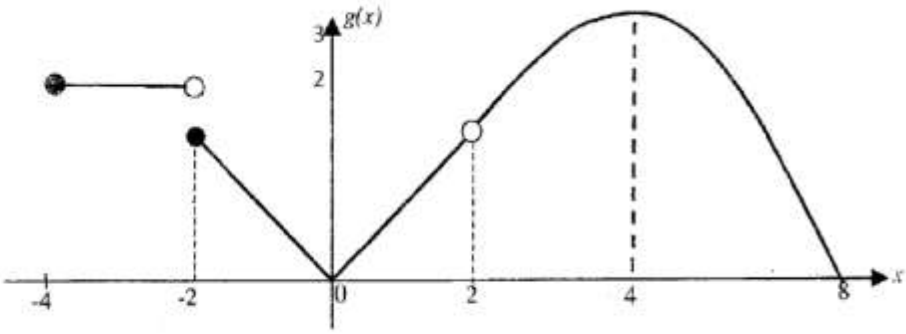
Visual aids

https://www.youtube.com/watch?v=JrYmEo6hoiU
https://www.youtube.com/watch?v=YOuiXpLqDr0
https://www.youtube.com/watch?v=fml0-ELYLaE

Exercises

Activities :

Exercise: 1 The graph of a function $g(x)$ is given below. Use the graph to answer the questions



Find the points where $g(x)$ is:

i.) *discontinuous*

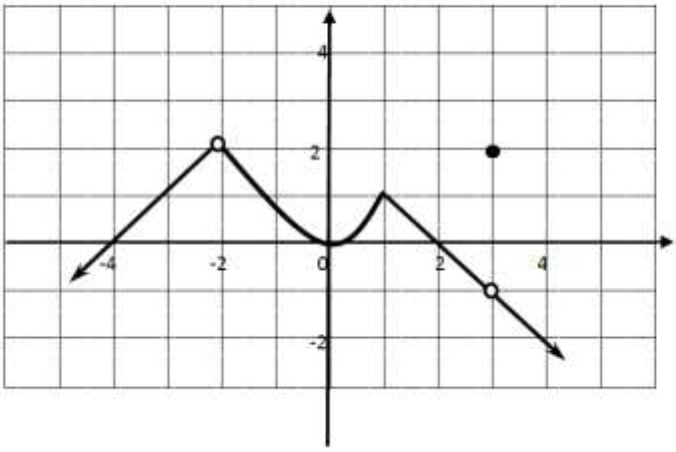
ii.) *non-differentiable*

iii.) $\lim_{x \to -2} g(x)$

iv.) $\lim_{x \to 2} g(x)$

<u>Exercise: 2</u>

The graph of a function $f(x)$ is given below. Use the graph to answer the questions that follow.



Find the places on the graph where $f(x)$ is:

i.) Discontinuous

ii.) Not differentiable

iii.) $\lim_{x \to -2^+} f(x)$

iv.) $\lim_{x \to 3} f(x)$

Assignment

Assessment

References

Barton, D., & Laird , S. (2002). *Delta Mathematics (Second Edition).* Pearson.

Lesson activity 2

<table>
<tr><td>
Teacher</td><td>Names :
- Mr Randal Taforua
- Miss Carla Lishi

Subject : MATHEMATICS WITH CALCULUS</td></tr>
<tr><td>
Date</td><td>Week 5
Wednesday 17th June, 2020</td></tr>
<tr><td></td><td>Strand 3: DIFFERENTIATION
Sub strand 3.1: Differentiation Basic Skills

Lesson Topic: Limits – Graphical and Numerical approach</td></tr>
<tr><td>
Learning outcomes</td><td>Students should be able to:
<table>
<tr><th>SUB-STRAND</th><th>SLO #</th><th>SPECIFIC LEARNING OUTCOME</th><th>SKILL LEVEL</th><th>SLO CODE</th><th>Achieved Yes/No</th></tr>
<tr><td>3.1</td><td>5</td><td>find limits of piecewise functions</td><td>1</td><td>Cal3.1.1.5</td><td></td></tr>
</table></td></tr>
<tr><td>
Introduction</td><td></td></tr>
<tr><td></td><td>Find your limit!</td></tr>
</table>

Learners notes

There are many ways or approaches that can be used to find limits. Listed below are the different types of methods and situations that entail this very subject.

1. **GRAPHICAL APPROACH**

   This is alluded to earlier in the previous section. The limit is obtained by approaching the x value from the left and right. If both approaches intersect at a point, then a LIMIT exists. The value of the limit is given by $f(a)$. [the corresponding y value, when x is substituted into f(x)]

2. NUMERICAL APPROACH [a.k.a the SANDWICH METHOD]

   This is done by substituting values that approach x on both terminal ends, and see what value it approaches.

   Example: Find the $\lim_{x \to 0} \frac{Sin\, x}{x}$

   At first glance, we know that this function is undefined because we cannot divide by "0". But if we substitute values approaching "0" at both ends, you will notice that it approaches 1.

x	0.1	0.01	0.001	0.0001	0	-0.0001	-0.001	-0.01	-0.1
$\frac{Sin\, x}{x}$	0.97	0.997	0.9998	0.9999		0.9999	0.9998	0.997	0.97

$$\lim_{x \to 0} \frac{Sin\, x}{x} \approx 1$$

3. **DIRECT SUBSTITUTION**

   The limits of any polynomial function can be calculated by direct substitution. Ie

   $\lim_{x \to 2} x^2 + 3 \; = (2)^2 + 3 = 4 + 3 = 7$

4. **LIMITS of RATIONAL FUNCTIONS**

   If the substitution of the value of "x" into the rational function gives an answer of $\frac{0}{0}$ then the limit may exist by factorizing the equation or simplifying the expression using an identity.

   $\lim_{x \to 3} \frac{x^2-9}{x-3} = \frac{(3)^2-9}{(3)-3} = \frac{0}{0}$ therefore the limit may exist; try by factorizing the numerator.

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

$$\lim_{x \to 3} \frac{(x + 3)(x - 3)}{x - 3}$$

$$\lim_{x \to 3} (x + 3)$$

$$(3) + 3 = 6$$

5. FINDING LIMITS TO INFINITY

When finding limits to infinity of certain rational functions, the following outcomes can be expected:

- **TOP HEAVY** (powers of x is more in the numerator)

  $\lim_{x \to \infty} x = \text{ limit is undefined or } \infty$

  Example:

  $$\lim_{x \to \infty} \frac{x^2 + 4x + 1}{x + 3} = \lim_{x \to \infty} \frac{x^2}{x} = \lim_{x \to \infty} x = \infty \text{ undefined.}$$

  (Only consider *highest power* in the numerator & denominator)

- **BOTTOM HEAVY** (powers of x is more in the denominator)

  $\lim_{x \to \infty} \frac{1}{x} = 0$ the limit is always ZERO.

  Example:

  $$\lim_{x \to \infty} \frac{x-4}{x^2-2x+3} = \lim_{x \to \infty} \frac{x}{x^2} = \lim_{x \to \infty} \frac{1}{x} = 0$$

- **BALANCED** (highest power in the numerator is the same as in the denominator)

  $\lim_{x \to \infty} = \frac{a}{b}$ the coefficients of the highest powers.

  Example:

  $$\lim_{x \to \infty} = \frac{3x^2+4x-1}{4x^2+6x-7} = \lim_{x \to \infty} \frac{3x^2}{4x^2} = \frac{3}{4}$$

6. SPECIAL CASE

The special case is when finding the limits of trig functions, to have an eye for the result :

$$\boxed{\lim_{x \to 0} \frac{Sin\ x}{x} \approx 1}$$

Example:

Find $\lim_{x \to 0} \frac{3\ Sin\ x}{4x} = \left(\frac{3}{4}\right) \lim_{x \to 0} \left(\frac{Sin\ x}{x}\right) = \left(\frac{3}{4}\right).(1) = \frac{3}{4}$

Visual aids	https://www.youtube.com/watch?v=iQmvdNN7FOc https://www.youtube.com/watch?v=M2BgBG2Su94 https://www.youtube.com/watch?v=sjLFl7Z8W_I
Exercises	**Activity :** Find the limits of the following: (All L1) (i) $\lim\limits_{x\to\infty}\frac{(x^3-2x)(x+3)}{(4x^2-1)(7x-2x^2)} = \lim\limits_{x\to\infty}\frac{x^4}{-8x^4} = \frac{1}{-8}$ (hint: Don't expand the whole thing. Only the ones with the highest powers in each bracket) (ii) $\lim\limits_{x\to3}\frac{2x^2-5x-3}{x-3}$ (iii) $\lim\limits_{h\to0}\frac{(2+h)^2-4}{h}$ (iv) $\lim\limits_{\theta\to0}\frac{Sin^2\theta}{1-Cos\theta}$ (v) $\lim\limits_{x\to\infty}\frac{2-y}{\sqrt{y+6y^2}}$ (vi) $\lim\limits_{x\to4}\frac{2-\sqrt{x}}{4-x}$ (vii) $\lim\limits_{x\to0}\frac{1-\cos^2 x}{x^2}$ (viii) $\lim\limits_{x\to\infty}\left(4-\frac{3}{x+1}\right)$
Assignment	
Assessment	Short quiz
References	Barton, D., & Laird , S. (2002). *Delta Mathematics (Second Edition).* Pearson.

Lesson activity 3

Teachers

Names :
- Mr Randal Taforua
- Miss Carla Lishi

Subject : MATHEMATICS WITH CALCULUS

Date

Week 5
Friday 19th June, 2020

Strand 3: DIFFERENTIATION
Sub strand 3.1: Differentiation Basic Skills

Lesson Topic: **Differentiating with the first principle**

Learning outcomes

Students should be able to:

SUB-STRAND	SLO #	SPECIFIC LEARNING OUTCOME	SKILL LEVEL	SLO CODE	Achieved Yes/No
3.3	**6**	**use the first principles to differentiate a function (only for polynomials of degree ≤ 3) using** $\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$	**3**	**Cal3.1.3.1**	

Introduction

This lesson focuses on how to differentiate polynomial functions of degree less or equal to 3 using the first principle.

It all starts with a first step!

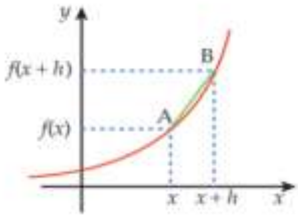
Learners notes

First Principle formula: $\boxed{f'(x) = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}}$

Background: This formula represents the tendency to find the gradient of a line, where h represents the horizontal distance and the gradient of the secant (a line joining two points, A and B on a curve) is given by:

$$m = \frac{difference\ in\ y-value}{difference\ in\ x-value} = \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$= \frac{f(x+h) - f(x)}{h}$$



How to use the Formula: $\boxed{f'(x) = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}}$

- Step 1 : Find an expression for $f(x+h)$ [**replace** "x" with $(x+h)$]
- Step 2 : Work out your numerator: ie **Subtract** $\boxed{f(x+h) - f(x)}$
- Step 3 : **Factorize** your answer in Step 2. [TRY and **single out** a " h".]
- Step 4 : Substitute your answer as the **numerator** into the formula. [Cancel " h".]
- Step 5 : Substitute zero into any remainders of " h" . This is your ANSWER.

Things to note:

- Your main GOAL is to cancel the "h" in the denominator. As long as the "h" lingers, the whole formula wil become undefined.
- Continue to write the stem " $\lim_{x \to 0}$ " in your steps, but remove it, the moment you substitute $h = 0$ into the equation.
- Step 2 and Step 3 is an ISOLATION clause, which is done to take away the congestion of working outside the formula. So when you substitute your working into the formula, you're dealing with a manageable expression and there is CLARITY in cancelling "h".

Example:

Use the first principle to differentiate the function $f(x) = x^2$

SOLUTION:

- Step 1: **Find an expression for $f(x+h)$:**

  This is done by substituting "$(x+h)$" where "x" is in the original equation.

  "$(x+h)$" $\quad f(x) = x^2 \quad \therefore f(x+h) = (x+h)^2$

$$= (x+h)(x+h)$$
$$= x^2 + xh + xh + h^2$$
$$= x^2 + 2xh + h^2$$

<table>
<tr><td></td><td>

- <u>Step 2</u>: Subtract $f(x+h) - f(x)$:

$$f(x+h) \quad - \quad f(x)$$
$$(\cancel{x^2} + 2xh + h^2) - \cancel{x^2}$$
$$\underline{= 2xh + h^2}$$

- <u>Step 3</u>: Factorize your answer in Step 2 (single out a "h" in the expression)

$$2xh + h^2 = h\,(2x+h)$$

- <u>Step 4</u>: Subtitute your answer in Step 3 as the <u>numerator</u> in the formula. (Cancel "h")

$$\lim_{h\to0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to0}\frac{\cancel{h}\,(2x+h)}{\cancel{h}} = \lim_{h\to0}(2x+h)$$

- <u>Step 5</u> : Subtitute $h = 0$ into the final expression. (you can REMOVE the STEM now)

$$\lim_{h\to0}(2x+h)$$
$$(2x+0)$$
$$\underline{2x}$$

</td></tr>
<tr><td>Visual aids</td><td>

https://www.youtube.com/watch?v=gxwbNdBlEMw
https://www.youtube.com/watch?v=cdisv5VksuY
https://www.youtube.com/watch?v=IGTcIUriGBs

</td></tr>
<tr><td>Exercises</td><td>

<u>Activity :</u>
Use the first principle to differentiate the following : (exercise ranging from simple – intermediate –complex) : (All L3)

i) $f(x) = 4x$
ii) $f(x) = 2x + 1$
iii) $f(x) = 3x^2$
iv) $f(x) = 2x^2 + 3x + 1$
v) $f(x) = \sqrt{x}$ *(Hint : step 3 : multiply by the conjugate of step 2 to cancel h)*

</td></tr>
<tr><td>Assignment</td><td></td></tr>
<tr><td>Assessment</td><td></td></tr>
<tr><td>References</td><td>

Barton, D., & Laird , S. (2002). *Delta Mathematics (Second Edition).* Pearson.

</td></tr>
</table>

Lesson activity 4 :

Teachers

Names :

- Mr Randal Taforua
- Miss Carla Lishi

Subject : MATHEMATICS WITH CALCULUS

Date

Week 5

Friday 19th June, 2020

Strand 3: DIFFERENTIATION

Sub strand 3.1: Differentiation Basic Skills

Lesson Topic: **Differentiating sum of functions**

Learning outcomes

Students should be able to:

SUB-STRAND	SLO #	SPECIFIC LEARNING OUTCOME	SKILL LEVEL	SLO CODE	Achieved Yes/No
3.3	7	**differentiate** sums of functions	2	Cal3.1.2.1	

Introduction

Learners notes

The derivative of a "Sum of functions" is equivalent to finding the sum of the derivative of its "**Individual components**". This can be the same to for a "*Difference*".

$$\boxed{[(f(a) + f(b)]' = f'(a) \pm f'(b)}$$

- **Polynomials**

$y = 2x^3 + x^2 + 4$ Find: y′. (hint: differentiate the *individual parts* in the equation)

$y' = (2x^3)' + (x^2)' + (4)'$

$y' = 6x^2 + 2x + 0$

$y' = 6x^2 + 2x$

- **Trigonometric Functions**

$Let\ f(x) = \sin x + \cos x\ .Find\ \ f'(x).$
$f'(x)\ =\ (\sin x)' + (\cos x)'$
$f'(x)\ = \cos x\ -\ \sin x$

Same can be said for the "*Difference*" or "*Subtraction*".

$Let\ f(x) = \sin x - \cos x\ .Find\ \ f'(x).$
$f'(x)\ =\ (\sin x)' - (\cos x)'$
$f'(x)\ = \cos x - (-\ \sin x)$
$f'(x)\ = \cos x +\ \sin x$

- **Combination of addition and subtraction.**

$Let\ f(x) =\ 2x^3 +\ 6x^2 -\ 10x - 9$. Find $f'(x)$

Solution:

Find the derivatives of each component:

- $(2x^3)' =\ 6x^2$
- $(6x^2)' =\ 12x$
- $(10x)' = 10$
- $(9)'\ \ = \ 0$

$\therefore f'(x) =\ 6x^2\ +\ 12x\ -\ 10\ -\ 0$
$\qquad\quad =\ 6x^2\ +\ 12x\ -\ 10$

Visual aids

https://www.youtube.com/watch?v=keflHzkhKyE
https://www.youtube.com/watch?v=PEqCa0U77mc

Exercises

Activity :

Differentiate the following: (All L2)

- Polynomials

i.) $f(x) =\ 3x^5 - 2x + 3$

ii.) $f(x) = \frac{2}{3}x^9\ +\ \ 6x\ \ - x^{-3}$

	iii.) $f(m) = -3m^2 - 5m^3$ iv.) $*f(s) = -\frac{2}{s^2} - \frac{3}{s^4}$ v.) $*y = \frac{2}{3}x^{\frac{1}{2}} - \frac{1}{3}x^6 + 23$ vi.) $*y = \sqrt{x} - \frac{3}{\sqrt[3]{x^4}}$ vii.) $*y = 5ax^{3a} - 7bx^{6b}$ $(differentiate\ with\ respect\ t$
Assignment	
Assessment	
References	Barton, D., & Laird , S. (2002). *Delta Mathematics (Second Edition).* Pearson.

Lesson activity 5 :

Teachers

Names :

- Mr Randal Taforua
- Miss Carla Lishi

Subject : MATHEMATICS WITH CALCULUS

Date

Week 6

Monday 22nd June, 2020

Strand 3: DIFFERENTIATION

Sub strand 3.1: Differentiation Basic Skills

Lesson Topic: **Quotient rule**

Learning outcomes

Students should be able to:

SUB-STRAND	SLO #	SPECIFIC LEARNING OUTCOME	SKILL LEVEL	SLO CODE	Achieved Yes/No
3.3	**8**	**differentiate** quotients, where f and g are both singular functions	**2**	**Cal3.1.2.2**	

Introduction

Do not fear failure but rather fear not **trying** !

Learners notes

The Quotient Rule is applied when finding the derivatives of a quotient of two functions in the form of $\frac{f(x)}{g(x)}$, whereby f and g are differentiable at x and $g(x) \neq 0$.

Quotient Rule

If $y = \left(\frac{f(x)}{g(x)}\right)$ then $y' = \frac{g(x).f'(x) - f(x).g'(x)}{(g(x))^2}$

In short we can say: $y = \frac{f}{g}$ $then\ y' = \frac{g.f' - f.g'}{g^2}$

Example:

Find y' if $y = \frac{2x^3}{x^2+1}$

$Let\ f = 2x^3 \quad \therefore \quad f' = 6x^2 \qquad\qquad Let\ g = x^2 + 1 \quad \therefore g' = 2x$

$y' = \frac{g.f' - f.g'}{g^2} = \frac{(x^2+1)(6x^2) - (2x^3)(2x)}{(x^2+1)^2} = \frac{6x^4 + 6x^2 - 4x^4}{(x^2+1)^2} = \frac{2x^4 + 6x^2}{(x^2+1)^2}$

An Alternative of doing the same problem so that the Quotient Rule is ***easier to cram***, is by finding the derivatives of each function and list the answers in columns – one underneath the other. ; ie

<u>Denominator</u> $g(x)$	<u>Numerator</u> $f(x)$
$g = x^2 + 1$	$f = 2x^3$
$\therefore\ g' = 2x$	$f' = 6x^2$

The arrangement above is tailor made to get the NUMERATOR in the Quotient Rule. The numerator can be calculated by "**cross-multiplying**" the expressions above.

$$\begin{array}{ll} g = x^2 + 1 & f = 2x^3 \\ \therefore\ g' = 2x & f' = 6x^2 \end{array}$$

$$y' = \frac{g.f' - f.g'}{g^2} = \frac{(x^2+1)(6x^2) - (2x^3)(2x)}{(x^2+1)^2}$$

There are some cases where the numerator will be hard to be simplified, so it could be left in their crude form.

Visual aids

https://www.youtube.com/watch?v=8jVDEcQ0wXk
https://www.youtube.com/watch?v=O6M4O7zY5eA

Exercises

<u>Activity :</u>

Use the Quotient Rule to differentiate the following: (All L2)

i.) $\boldsymbol{y = \frac{3x - 1}{4x + 2}}$

ii.) $\boldsymbol{y = \frac{3x^5}{x+1}}$

Assignment

Assessment	
References	Barton, D., & Laird , S. (2002). *Delta Mathematics (Second Edition).* Pearson.

Lesson activity 6

<table>
<tr><td>
Teachers</td><td>Names :
- Mr Randal Taforua
- Miss Carla Lishi
Subject : MATHEMATICS WITH CALCULUS</td></tr>
<tr><td>
Date</td><td>Week 6
Tuesday 23rd June, 2020
Wednesday 24th June, 2020</td></tr>
<tr><td></td><td>Strand 3: DIFFERENTIATION
Sub strand 3.1: Differentiation Basic Skills

Lesson Topic: Chain rule</td></tr>
<tr><td>
Learning outcomes</td><td>Students should be able to:
<table>
<tr><th>SUB-STRAND</th><th>SLO #</th><th>SPECIFIC LEARNING OUTCOME</th><th>SKILL LEVEL</th><th>SLO CODE</th><th>Achieved Yes/No</th></tr>
<tr><td>3.3</td><td>9</td><td>differentiate composite functions (chain rule), where f and g are both singular functions</td><td>2</td><td>Cal3.1.2.3</td><td></td></tr>
</table></td></tr>
<tr><td>
Introduction</td><td></td></tr>
<tr><td></td><td>Catch phrase for the lesson</td></tr>
</table>

Learners notes

Background

A Composite Function can be defined as having two functions: f and g, whereby one is logged in as an INTERNAL function, and the other as an EXTERNAL one. ie $(f\ o\ g)(x) = f(g(x))$.

Example:

- If $f(x) = x^2$ and $g(x) = x + 1$ then $(f\ o\ g)(x) = (x + 1)^2$

  The inner function is $g(x) = x + 1$ and the outer function is $f(x) = x^2$
- If $y = (3x^3 + x^2)^4$
  The inner function is $g = (3x^3 + x^2)$ and the outer function is $y = x^4$

Formula for Differentiating a Composite Function

$$\boxed{If\ y = f(g(x))\ then\ y' = f'(g(x)) \cdot g'(x)}$$

Example:

Differentiate $y = (x^2 + 4x)^3$

Solution

External : $y = x^3$ $\quad y' = 3x^2$ $\quad$ replace x $with\ the\ internal\ function\ ie$ $y = 3(x^2 + 4x)^2$

Internal : $g(x) = x^2 + 4x$ $\quad g'(x) = 2x + 4$

Using the FORMULA: $\boxed{y' = f'(g(x)) \cdot g'(x)}$ $=$ $3(x^2 + 4x)^2.(2x + 4)$

Alternative working is to use the Leibniz notation: where $y = f(g(x))$ and $u = g(x)$

Where the $\boxed{\text{CHAIN RULE : } \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}}$

$Let\ u = x^2 + 4x$ $\quad \therefore \frac{du}{dx} = 2x + 4$ $\qquad Let\ y = u^3$ $\quad \therefore \frac{dy}{du} = 3u^2$

Using the CHAIN RULE : $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \rightarrow \frac{dy}{dx} = 3u^2.(2x + 4) = 3(x^2 + 4x)^2.(2x + 4)$

Alternative Analogy:

Is to differentiate the equation in **LAYERS**. Like an onion, each layer is stripped from the outer – innermost.

- The outer layer will be the **POWER**.
- The inner layer can be a **function** or **Trig function**
- The innermost layer can be an **ANGLE** (if your 2[nd] Layer was a Trig Function)

Example : (2 layers)

Differentiate: $y = (2x + 5)^3$

Solution:

Identify the layers.....starting from the *"outside"* into the *"innermost"*. ie,

<table>
<tr>
<td></td>
<td>

$y = (x^2+5)^3$ 1. Power

2. Inner Function

$\frac{dy}{dx} = (derivative\ power)\ .\ (derivative\ funtion)$

$\frac{dy}{dx} = 3(x^2+5)^2 \quad . \quad (2x) \quad = \quad 6x\,(x^2+5)^2$

<u>Example : (3 Layers)</u>

Differentiate $y = Sin^3(2x+1)$

<u>Solution</u>:

This equation can be rewritten as : $y = [Sin\,(2x+1)]^3$.

Identify the Layers.

1. Power

$y = [Sin\,(2x+1)]^3$

2. Trig Function 3. Angle

Note: When differentiating Trig Functions, the ANGLE will NEVER CHANGE.

$y' = (derivative\ of\ Power)\ .\ (derivative\ of\ Trig\ Function)\ .\ (derivative\ of\ Angle)$

$y' = 3[Sin\,(2x+1]^2 \quad . \quad Cos\,(2x+1) \quad . \quad (2)$

$y' = 6\ Sin^2(2x+1)\ .\ Cos\,(2x+1)$

[Notice: angles have NOT changed. Don't be tempted to expand the angle with coefficients.]

</td>
</tr>
<tr>
<td>Visual aids</td>
<td>

https://www.youtube.com/watch?v=HaHsqDjWMLU
https://www.youtube.com/watch?v=KKaRHdZ-Qus

</td>
</tr>
<tr>
<td>Exercises</td>
<td>

<u>Activity: use the chain rule or the formula for composite functions to differentiate the following.</u>

i) $y = (2x+3)^7$

ii) $y = (8x+x^3)^5$

</td>
</tr>
</table>

	iii.) • $y = \sqrt{x^2 + 3x + 1} = (x^2 + 3x + 1)^{\frac{1}{2}}$ iv.) • $y = \frac{-2}{\sqrt{2x - x^3}} = \frac{-2}{(2x - x^3)^{\frac{1}{2}}} = -2(2x - x^3)^{-\frac{1}{2}}$ v.) • $y = \left(\frac{x + 1}{3x - 1}\right)^3$
Assignment	
Assessment	Short quiz
References	Barton, D., & Laird , S. (2002). *Delta Mathematics (Second Edition).* Pearson.

Lesson activity 7

Teachers

Names :
- Mr Randal Taforua
- Miss Carla Lishi

Subject : MATHEMATICS WITH CALCULUS

Date

Week 6
Thursday 25th June, 2020
Friday 26th June, 2020

Strand 3: DIFFERENTIATION
Sub strand 3.1: Differentiation Basic Skills

Lesson Topic: **Differentiating parametric functions**

Learning outcomes

Students should be able to:

SUB-STRAND	SLO #	SPECIFIC LEARNING OUTCOME	SKILL LEVEL	SLO CODE	Achieved Yes/No
3.3	10	**differentiate** parametric functions	2	Cal3.1.2.4	

Introduction

Parametric equations are functions that are written with respect to another variable.

Practice makes perfect!

Learners notes

Introduction

Parametric equations are functions that are written with *respect to another variable. Often this third variable can come in terms of time, "t" or with "θ" when dealing with trig functions.* Eg ; $x = 5 + 3t$; $y = \ Sin\theta$

So when told to differentiate such functions, each *individual* function is differentiated with respect to the 3rd variable. When finding $\frac{dy}{dx}$, the CHAIN RULE is applied:

By using the **CHAIN RULE**; we can get $\frac{dy}{dx}$ by dividing the two derivatives; ie

$$\boxed{\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dt} \div \frac{dx}{dt} \\ \frac{dy}{dx} &= \frac{dy}{\cancel{dt}} \times \frac{\cancel{dt}}{dx} \end{aligned}}$$

Example: 1

Find $\frac{dy}{dx}$ if $x = 4t + 3$ and $y = 8t^2 + 2t^3$

Solution:

Step: 1 Find the individual derivatives. (*divide them with respect to t*)

$x = 4t + 3 \quad \therefore \frac{dx}{dt} = 4$ $\qquad\qquad$ $y = 8t^2 + 2t^3 \quad \therefore \frac{dy}{dt} = 16t + 6t^2$

Step:2 Divide the derivatives:

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} \qquad \therefore \frac{dy}{dx} = \frac{16t + 6t^2}{4}$

Step: 3 Simplify the expression by splitting the denominator.

$$\frac{\mathbf{dy}}{\mathbf{dx}} = \frac{16t + 6t^2}{\mathbf{4}} = \frac{\mathbf{16t}}{\mathbf{4}} + \frac{\mathbf{6t^2}}{\mathbf{4}} = \mathbf{4t} + \frac{\mathbf{3t}}{\mathbf{2}}$$

Example : 2

Find $\frac{dy}{dx}$ if $y = 4\,Sin\,\theta$ and $x = cos^2\theta$

Step: 1 Find the individual derivatives. (*divide them with respect to θ*)

$y = 4\,Sin\,è \quad \therefore \frac{dy}{d\theta} = 4\,Cos\,\theta$ $\qquad\qquad$ $x = cos^2\theta = [Cos\,\theta]^2 \quad \therefore \frac{dx}{d\theta}$

$= (power)'.(trig\ fun)'$

$= 2\,[Cos\,\theta]\,.-Sin\,\theta$

$= -2Cos\theta.Sin\,\theta$

Step:2 Divide the derivatives:

$\frac{dy}{d\theta} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} \qquad \therefore \frac{dy}{dx} = \frac{4\,Cos\,\theta}{-2Cos\theta.Sin\,\theta}$

Step: 3 Simplify the expression by cancelling

$$\therefore \frac{\mathbf{dy}}{\mathbf{dx}} = \frac{4\,Cos\,\theta}{-2Cos\theta.Sin\,\theta} = \frac{\mathbf{-2}}{\mathbf{Sin\theta}}$$

Visual aids

https://www.youtube.com/watch?v=kf2dZWqLnqE
https://www.youtube.com/watch?v=hljazIjVnz8

Exercises

Activity :

	Find $\frac{dy}{dx}$ for the following parametric equations: (All L2) i.) $x = t^2 \quad and \quad y = 4t$ ii.) $x = t^2 \quad and \quad y = t^2 + 8t + 3$ iii.) $x = t^2 + 1 \quad and \quad y = 2t^2 + 6t$ iv.) $x = 3sin\theta + 2 \ and \quad y = 4\cos\theta - 1$ v.) $x = 2cos\theta \quad and \ y = 2sin^2\theta$
Assignment	
Assessment	Short quiz
References	Barton, D., & Laird , S. (2002). *Delta Mathematics (Second Edition).* Pearson.

Lesson activity 8

Teacher

Names :

- Mr Randal Taforua
- Miss Carla Lishi

Subject : MATHEMATICS WITH CALCULUS

F

Week 7
Monday 29th June, 2020
Tuesday 30th June, 2020

Strand 3: DIFFERENTIATION
Sub strand 3.1: Differentiation Basic Skills

Lesson Topic: **Second derivatives**

Learning outcomes

Students should be able to:

SUB-STRAND	SLO #	SPECIFIC LEARNING OUTCOME	SKILL LEVEL	SLO CODE	Achieved Yes/No
3.3	11	**finding** the second derivatives of a given function	2	Cal3.1.2.5	

Introduction

Learners notes

Introduction

Finding the second derivative simply means to differentiate the *derived* function. It means having "*back-to-back*" differentiations after differentiating the original function. We differentiate a function first, then our answer is differentiated again. This can be shown by using various notations:

$$f(x) \quad differentiate\ once \ \rightarrow \ f'(x) \quad differentiate\ twice \ \rightarrow \ f''(x)$$

$$y \quad differentiate\ once \ \rightarrow \ \frac{dy}{dx} \quad differentiate\ twice \ \rightarrow \ \frac{d^2y}{dx}$$

Example:

Determine the second derivative for the function; $\boldsymbol{f(x) = x^3 + 3x^2 + 2x + 3}$.

- 1st Derivative: Differentiate the original equation.
  $Differentiate\ f(x) \quad : \quad f'(x) = 3x^2 + 6x + 2$

- 2nd Derivative: Differentiate your answer in Step:1
  $Differentiate\ f'(x) \quad : \quad f''(x) = 6x + 6$

	Find $\frac{d^2y}{dx}$ if $y = Cos\, x$ • 1[st] Derivative: Differentiate the original equation. $Differentiate\ y \quad : \frac{dy}{dx} = -Sin\, x$ • 2[nd] Derivative: Differentiate your answer in Step:1 $Differentiate\ \frac{dy}{dx} : \frac{d^2y}{dx} = -Cos\, x.$
Visual aids	https://www.youtube.com/watch?v=WC5VYKI807Q
Exercises	**Activity :** Find the second derivative of the following: (All L2) i.) $y = 8x^3 + 5x$ ii.) $f(x) = 2x^3 + 6x^2 + 2x + 1$ iii.) $Evaluate\ f''(1)\ if\ f(x) = 5x^3$ iv.) $Find\ \frac{d^2y}{dt}\ if\ y = t^3 + 4t^2 - 2$ Workout the second derivative of the following functions: **i.)** e^{3x} **ii.)** $ln(5x)$ iii.) $Sin\ 6x$ iv.) $2\ Cos\ 3t$

 Assignment	
 Assessment	Short Quiz (20 mins)
 References	Barton, D., & Laird , S. (2002). *Delta Mathematics (Second Edition).* Pearson.

Lesson activity 9

<table>
<tr><td>
Teacher</td><td>Names :
- Mr Randal Taforua
- Miss Carla Lishi

Subject : MATHEMATICS WITH CALCULUS</td></tr>
<tr><td>
Date</td><td>Week 7
Wednesday 1st July, 2020
Thursday 2nd July, 2020</td></tr>
<tr><td></td><td>Strand 3: DIFFERENTIATION
Sub strand 3.1: Differentiation Basic Skills

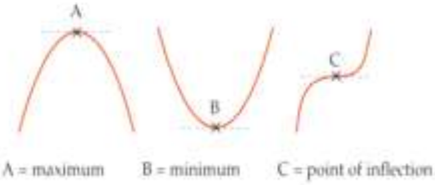
Lesson Topic: Differentiate to determine the maxima and minima</td></tr>
<tr><td>
Learning outcomes</td><td>Students should be able to:
<table>
<tr><th>SUB-STRAND</th><th>SLO #</th><th>SPECIFIC LEARNING OUTCOME</th><th>SKILL LEVEL</th><th>SLO CODE</th><th>Achieved Yes/No</th></tr>
<tr><td>3.3</td><td>12</td><td>solve problems by finding the maxima or minima with proof for polynomial and rational functions</td><td>3</td><td>Cal3.1.3.2</td><td></td></tr>
</table></td></tr>
<tr><td>
Introduction</td><td></td></tr>
<tr><td></td><td>Never give up, try again, try harder!</td></tr>
</table>

Learners notes

Background

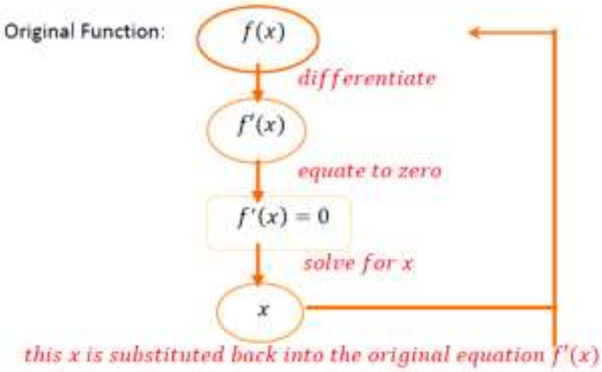
The *maximum* and *minimum* points of a function can be solved by equating $f'(x) = 0$ and finding the value of "x". There are many ways in which maxima or minima questions can be phrased, but they all stand for the same thing: for example: $Maxima \approx Minima \approx stationary\ points \approx turning\ points$

- The stationary point is where the gradient is zero.
- Stationary point can be found by: $f'(x) = 0$
- The Nature of each stationary point can be found by finding the second derivative of $f(x)$:
  - $f''(x) < 0$: $maximum$
  - $f''(x) > 0$: $minimum$
  - $f''(x) = 0$: $point\ of\ inflection$



The dynamics behind finding the maximum/minimum points can be followed by using this basic flow chart:

FLOWCHART for MAXIMA / MINIMA



Example

Find the coordinates of the *stationary points* for $y = x^2(x-6)$ and determine the *nature* of each.

Solution

$$\boxed{y = x^2(x-6)}$$
$$= x^3 - 6x^2$$
$$\boxed{y' = 3x^2 - 12x}$$
$$= 3x(x-4)$$
$$\boxed{0 = 3x(x-4)}$$

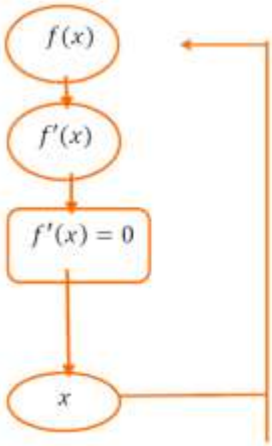
$0 = 3x$ $\qquad$ $0 = (x-4)$

$\frac{0}{3} = x$ $\qquad$ $0 + 4 = x$

$\boxed{0 = x}$ $\qquad$ $\boxed{4 = x}$

Substituting "x" back into original eqn

FLOWCHART



$When\ x = 0$: $y = 0^2(0-6) = 0$ $\qquad$ $When\ x = 4$: $y = 4^2(4-6) = 16(-2) = -32$

$\therefore (0, 0)$ $\qquad$ $\therefore (4, -32)$

NATURE

Finding the NATURE of each stationary point can be done by finding the second derivative $\frac{d^2y}{dx}$

- $y' = 3x^2 - 12x \quad \therefore\ y'' = 6x - 12$

At point $(0\ ,0)$, we substitute the x value of stationary point into the second derivative:

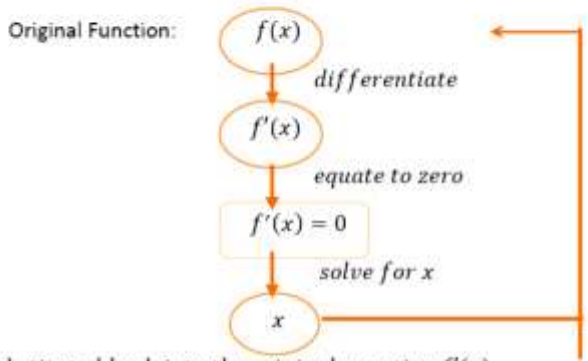
$\therefore\ y'' = 6x - 12 \ = 6(0) - 12 \ = 0 - 12 \ = -12$ since it is NEGATIVE $f''(x) < 0$
$\therefore$ *the staionary point* $(0{,}0)$ *is a MAXIMUM.*

At point $(4\ ,-32)$, we substitute the x value of stationary point into the second derivative:

$\therefore\ y'' = 6x - 12 \ = 6(4) - 12 \ = 24 - 12 \ = 12$ since it is POSITIVE $f''(x) > 0$
$\therefore$ *the staionary point* $(4,-32)$ *is a MINIMUM*

Word Problems in Maxima/Minima

The concept of finding the Maxima/Minima, is very similar to the concept of finding the stationary points as stipulated in the previous exercises, hence, the same flowchart can be used to solve word problems. The principle steps are shown by the flow chart given below:



this x is substituted back into the original equation $f'(x)$

CHALLENGE:

The only difficulty here is that the ORIGINAL equation is NOT given. You will have to use the information given and your basic knowledge of some common formulas like Area, Volume, Surface Area, Scale Factors, etc depending on the type of question being asked.

TIPS FOR ATTEMPTING MAXIMA/MINIMA PROBLEMS

- Read the word problem twice (or thrice), and highlight specific "terms" the problem uses.
- Find out what is it asking for? Area, Volume....and what shape are we dealing with, a sphere/triangle, etc?
- Draw a diagram. Try and have a mental picture of what you're solving.
- Use the information given and the formulas anticipated to find your EQUATION.
- Once you have your equation – use the CONCEPT MAP above to get your MAXIMA/MINIMA.

Example

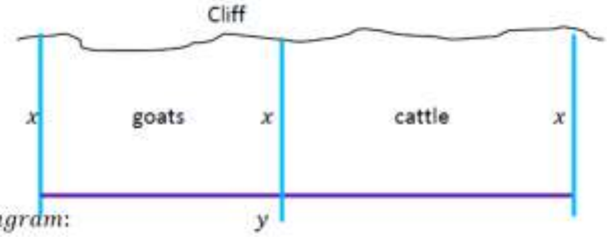
A farmer wanted to fence a rectangular piece of land for his cattle and goats. The rectangular paddock is to be partitioned to cater for both animals. The land is exactly located to the side of a cliff. If he used 1200m of wire, to fence his land, find the maximum area that land will enclose?

Step : 1

Key words: "rectangular" – so the shape is a rectangle. *Maximum AREA : Area = L x W*

Step : 2

Draw a diagram:



Label the sides in the diagram:

Note: the one with many sides – label as x and the one as "y"

Step : 3

Use the information to come up with an equation:

"he used 1200m of wire, to fence his land" $x + x + x + y = 1200 \quad \therefore \quad y + 3x = 1200$

$$y = 1200 - 3x$$

Step : 4 Use the information to find your Original Formula

$Area = length \ x \ width \rightarrow A = x \cdot y \rightarrow A = x(1200 - 3x) \therefore \boxed{A = 1200x - 3x^2}$

$A = 1200x - 3x^2$

$$A' = 1200 - 6x \qquad \boxed{A'(x)}$$

$$0 = 1200 - 6x \qquad \boxed{A'(x) = 0}$$

$$6x = 1200$$

$$x = \frac{1200}{6}$$

$$\mathbf{x = 200} \qquad x$$

Substituting into the Area Formula : $A = 1200(200) - 3(200)^2$

$$\mathbf{A = 120,000m^2}$$

Visual aids

https://www.youtube.com/watch?v=pvLj1s7SOtk
https://www.youtube.com/watch?v=pInFesXIfg8

Exercises

Activity :

1. (All L3)

1. Find the stationary points for the following and determine the nature of each point:

i.) $y = 2x^3 - 3x^2 - 12x$

ii.) $y = \frac{x^2 - 4}{x+1}$

iii.) $y = \frac{x^2}{x - 1}$

	2. A square piece of tin has 12 *cm* on each side. An open box is formed by cutting out equal square pieces at the corners and bending upward the projecting portions which remain. Find the maximum volume that can be obtained. 3. The sum of one number and twice another is 24. Find the two numbers so that their *product* is a maximum 4. Find the stationary points for the following, and determine their nature. a.) $y = 2x - \frac{1}{6}x^3$ b.) $f(x) = x^2 + 3x + 4$ c.) $y = cosx + x \quad for \quad 0 < x < \pi$
Assignment	
Assessment	Short quiz
References	Barton, D., & Laird , S. (2002). *Delta Mathematics (Second Edition).* Pearson.

Lesson activity 10

Teachers

Names :

- Mr Randal Taforua
- Miss Carla Lishi

Subject : MATHEMATICS WITH CALCULUS

Date

Week 7
Friday 3rd July, 2020

Strand 3: DIFFERENTIATION
Sub strand 3.1: Differentiation Basic Skills

Lesson Topic: **Points of inflection**

Learning outcomes

Students should be able to:

SUB-STRAND	SLO #	SPECIFIC LEARNING OUTCOME	SKILL LEVEL	SLO CODE	Achieved Yes/No
3.3	**13**	**differentiate** to find the points of inflection	**3**	**Cal3.1.3.3**	

Introduction

Catch phrase for the lesson

Learners notes

Recall:

Stationary points and points of inflection.

A **stationary point** on a curve is one where the gradient is zero. Stationary points include not only turning points (maximum points and minimum points), but also a new kind of point called **a point of inflection**.

One way to determine the point of inflection is to see its point of CONCAVITY.

Concavity tells us whether the graph bends downwards or upwards as shown in the diagram below.

C = point of inflection

$\boxed{f''(x) < 0}$

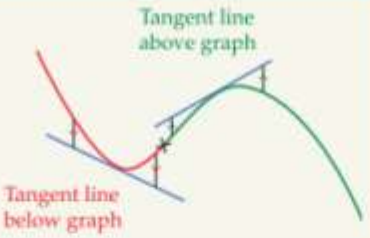
This graph is concave up:

$\boxed{f''(x) > 0}$

This graph is concave down:

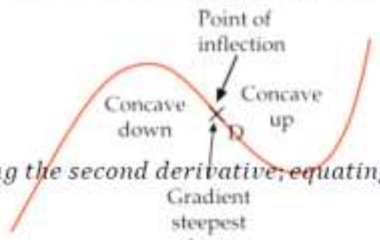
A function, $f(x)$, is **concave up** on an interval $a < x < b$ if its graph lies *above* its tangent line at any point in the interval.

A function, $f(x)$, is **concave down** on an interval $a < x < b$ if its graph lies *below* its tangent line at any point in the interval.



(*Source: Anton, Calculus*)

The POINT in INFLECTION exist where there is a change in concavity.



$\boxed{f''(x) = 0}$

$finding\ the\ second\ derivative; equating\ x = 0$

<u>Example</u>:

For the function, find all points of inflection or determine that no such points exist.

$$f(x) = \frac{\ln x}{x} \text{ for } x > 0$$

Answer

$$f(x) = \frac{\ln x}{x} \text{ for } x > 0$$

We use the quotient rule a couple of times to find the first two derivatives:

$$\begin{aligned} f'(x) &= \frac{\frac{1}{x} \cdot x - \ln x(1)}{x^2} \\ &= \frac{1 - \ln x}{x^2} \\ f''(x) &= \frac{-\frac{1}{x} \cdot x^2 - (1 - \ln x)(2x)}{x^4} \\ &= \frac{-x - (2x - 2x \ln x)}{x^4} \\ &= \frac{-3x + 2x \ln x}{x^4} \\ &= \frac{x(-3 + 2 \ln x)}{x^4} \\ &= \frac{-3 + 2 \ln x}{x^3} \end{aligned}$$

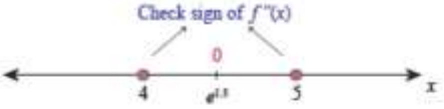
(Source:www.Shmoop.com)

The second derivative is undefined when $x = 0$, but that's not in the domain of f anyway sc doesn't count. The second derivative is 0 when

$$\begin{aligned} 0 &= -3 + 2 \ln x \\ 3 &= 2 \ln x \\ \frac{3}{2} &= \ln x \\ e^{3/2} &= x \end{aligned}$$

To confirm that this is an inflection point, we need to check what the sign of f'' does as we cross $x = e^{3/2}$ ≈ 4.48.

The numbers $x = 4$ and $x = 5$ are nice ones to use.

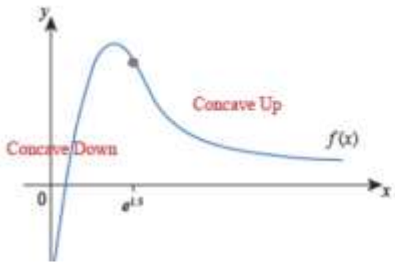


$$
\begin{aligned}
f''(4) &= \frac{-3+2\ln 4}{4^3} \\
&\approx -.00355 \\
&< 0 \\
f''(5) &= \frac{-3+2\ln 5}{5^3} \\
&\approx .00175 \\
&> 0
\end{aligned}
$$

Since the sign of f'' does change, $x = e^{3/2}$ is indeed an IP.

Sign of $f''(x)$

From the graph, this is believable:



EXAMPLE

For the function, find all points of inflection or determine that no such points exist.

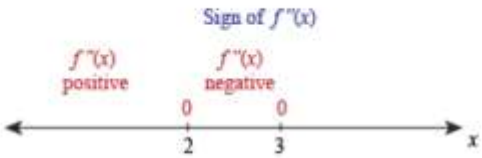
$$f(x) = \frac{1}{12}x^4 - \frac{5}{6}x^3 + 3x^2 + \frac{1}{2}$$

Answer

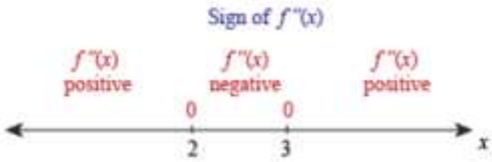
$$f(x) = \frac{1}{12}x^4 - \frac{5}{6}x^3 + 3x^2 + \frac{1}{2}$$

$$
\begin{aligned}
f'(x) &= \frac{4}{12}x^3 - \frac{3\cdot 5}{6}x^2 + 6x + 0 \\
&= \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x \\
f''(x) &= \frac{3}{3}x^2 - \frac{10}{2}x + 6 \\
&= x^2 - 5x + 6 \\
&= (x-2)(x-3).
\end{aligned}
$$

When x is between 2 and 3, the quantity $x - 2$ is positive but $x - 3$ is negative so f'' is negative:



Finally, when x is greater than 3 both $x - 2$ and $x - 3$ are positive, so f'' is positive:



Since the sign of f'' changes at $x = 2$ and $x = 3$, these are both inflection points.

Visual aids

https://www.youtube.com/watch?v=3TJLOCYrTes
https://www.youtube.com/watch?v=quA0wJ71GkQ
https://www.youtube.com/watch?v=k7hIb5ydWGc
https://www.youtube.com/watch?v=osOepf7K1gM

Exercises

Activity :

i.) For the function, find all points of inflection or determine that no such points exist.

$$f(x) = \frac{x^3 + 3}{2x^2}$$

ii.) For the function, find all points of inflection or determine that no such points exis

$$f(x) = \frac{1}{12}x^4 - \frac{5}{6}x^3 + 3x^2 + \frac{1}{2}$$

iii.) For the function, find all points of inflection or determine that no such points exis

$f(x) = x \ln x$ for $x > 0$

Source:www.Shmoop.com)

Assignment

Assessment

Barton, D., & Laird , S. (2002). *Delta Mathematics (Second Edition).* Pearson.

References	

WEEKLY CHECKLIST For Parents:

Term: 2 Week number 1 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 2 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 3 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 4 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 5 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 6 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 7 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 8 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 9 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 10 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 11 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 12 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				

Term: 2 Week number 13 Date…… to…… Month: …………

Subject	Number of lessons	Days	Tick when activity is complete	Parents comment	Signature
	1				
	2				
	3				
	4				
	5				
	6				