

# Topic 3: Statistics and Probability

## Collecting and Graphing Data

Data is information that has been collected. To find out what the data represents or tells us, we need to first put it into a **frequency table**.

**Frequency** tells us how many of a certain result or piece of information have been collected.

### *Example*

30 students did a short maths test that was marked out of 10. Here are their results:

7 9 8 8 4 5 7 9 6 6  
6 8 5 7 6 6 5 6 4 7  
5 6 7 6 5 8 6 7 4 6

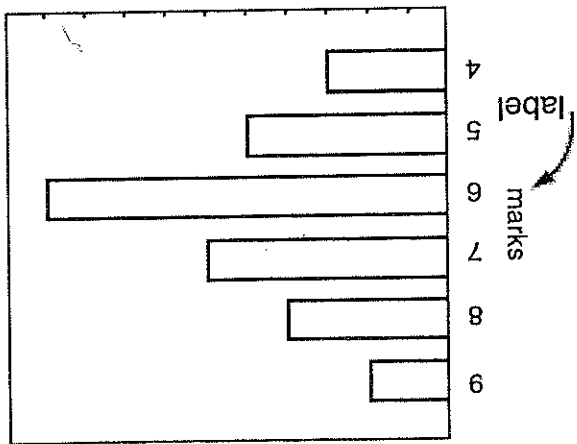
The frequency table would look like this:

Mark	Tally	Frequency
9	II	2
8	IIII	4
7	IIII I	6
6	IIII II	10
5	IIII	5
4	III	3
		<hr/> <hr/> 30

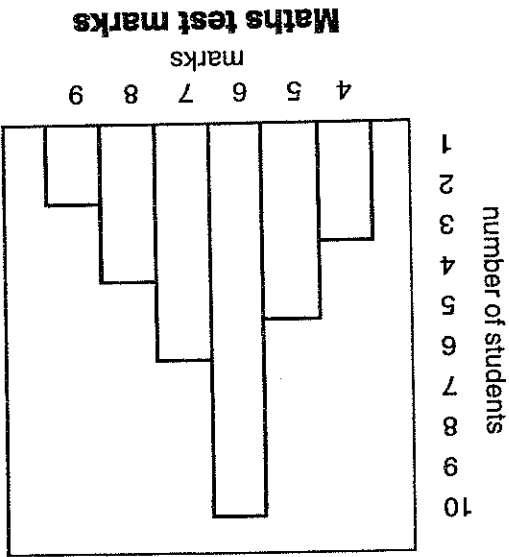
This information could be shown using a variety of graphs.

Here is the data from the table above used in different types of graphs.

## Bar Graphs



A special sort of bar graph has the columns touching. It is called a **histogram**.



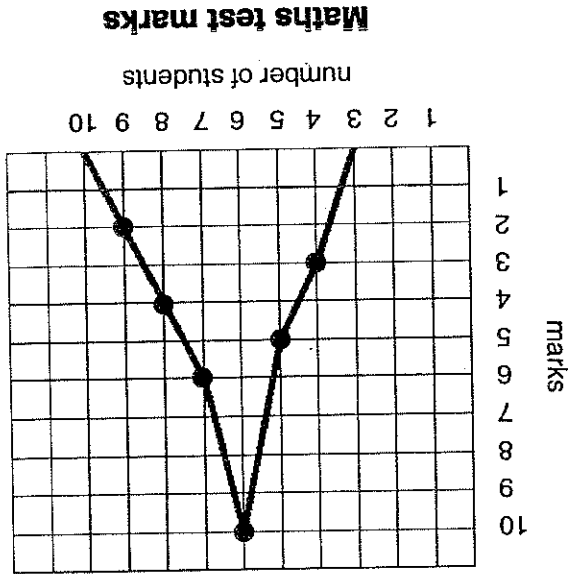
**Maths test marks** label  
 title  
 number of students

It is very important that all graphs have a **title** and that the axes are labelled.



## Line Graphs

Line graphs need to start and end at zero in this case, since no students scored 3 or 10.



## Pictographs

Pictographs are really another kind of table except the frequency is shown by pictures

Here  $\text{stick figure} = 2$  students so if we have a frequency of 3, we need to use half a picture.

$$\begin{aligned} 3 &= 2 + 1 \\ &= \text{stick figure} + \frac{1}{2} \text{ stick figure} \end{aligned}$$

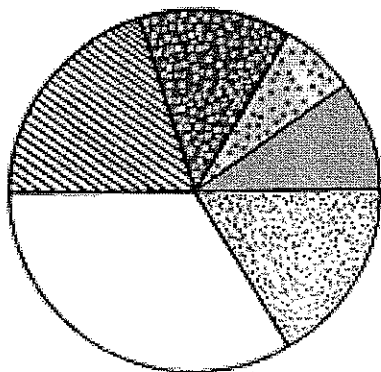
marks	
9	$\text{stick figure}$
8	$\text{stick figure} \text{ stick figure}$
7	$\text{stick figure} \text{ stick figure} \text{ stick figure}$
6	$\text{stick figure} \text{ stick figure} \text{ stick figure} \text{ stick figure} \text{ stick figure}$
5	$\text{stick figure} \text{ stick figure} \frac{1}{2} \text{ stick figure}$
4	$\text{stick figure} \frac{1}{2} \text{ stick figure}$

$\text{stick figure} = 2$  students

### Maths test marks

## Pie Graphs

To find the angles of the pie graph:



Maths test marks

$$\begin{aligned} 9 \text{ marks: } & \frac{2 \text{ students}}{30 \text{ students}} \times 360^\circ \\ & = \frac{2}{30} \times 360^\circ \\ & = 24^\circ \end{aligned}$$

$$\begin{aligned} 8 \text{ marks: } & \frac{4}{30} \times 360^\circ \\ & = 48^\circ \end{aligned}$$

$$\begin{aligned} 7 \text{ marks: } & \frac{6}{30} \times 360^\circ \\ & = 72^\circ \end{aligned}$$

$$\begin{aligned} 6 \text{ marks: } & \frac{10}{30} \times 360^\circ \\ & = 120^\circ \end{aligned}$$

$$\begin{aligned} 5 \text{ marks: } & \frac{5}{30} \times 360^\circ \\ & = 60^\circ \end{aligned}$$

$$\begin{aligned} 4 \text{ marks: } & \frac{3}{30} \times 360^\circ \\ & = 36^\circ \end{aligned}$$

**Exercise 3.1**

1. Graph the following data using bar graphs:

(a) Life span

Animal	Years
Crocodile	13
Elephant	60
Lion	25
Tiger	11
Hippopotamus	40
Zebra	22
Ostrich	58
Monkey	15

Country	Years
Australia	75
Mexico	64
Chad	32
Bangladesh	55
Singapore	72
Russia	69
Afghanistan	37
India	46

(b) Life expectancy

2. Graph the following data using pictographs:

(a) Favourite colour

Colour	Frequency
Red	4
Black	10
Green	4
Blue	9
White	3

(b) Favourite food

Food	Frequency
Chicken	7
Tuluk	5
Lap Lap	8
Manioc	1
Beef	4

3. Graph the following data using line graphs:

(a) Maths test results

Score	Students
24	1
25	4
26	6
27	7
28	6
29	3
30	2
31	1

(b) Spelling errors

Errors	Pages
0	1
1	2
2	5
3	6
4	7
5	4
6	2
7	1

4. Graph the following data using pie graphs:

(a) Favourite subject

Subject	Frequency
Science	10
English	6
Maths	12
Social Science	3
French	5

(b) Age of Year 9 students

Age	Frequency
13	2
14	17
15	14
16	7

5. For the data below, draw up a frequency table and then draw a bar graph:

**Number of matches in match boxes**

52 48 50 51 52 47 49 50  
 49 53 50 47 49 48 51 52  
 49 52 50 48 50 50 52 53  
 49 49 52 53 50 49

6. For the data below, make a frequency table and then draw a line graph:

**Number of students in a class**

27 22 24 21 19 26 25 20 25 24  
 21 21 25 18 23 24 25 27 22 23  
 19 20 24 22 26 24 21 22 24 18  
 22 25 21 24 19 26 20 22 26 21

7. For the data below, make a frequency table and then draw a pictograph:

**Number of children in a family**

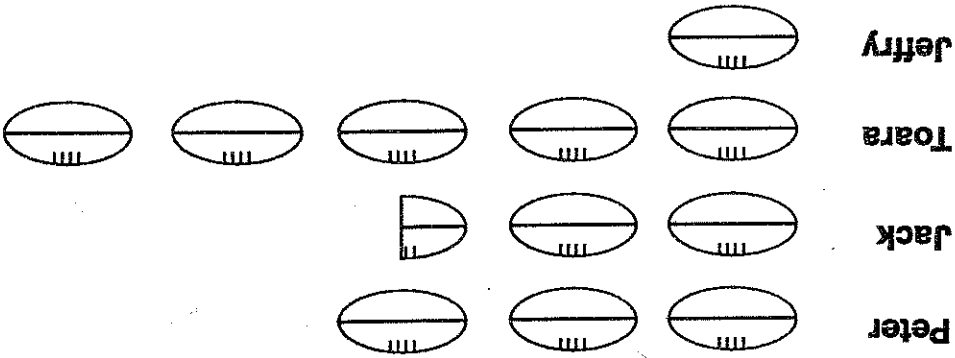
1 5 3 2 2 3 0 1 1 2 2 4 0  
 4 1 2 3 1 4 1 2 1 5 6 3 1  
 0 3 0 2 4 2 1 0 3 2 0 3 3  
 2 2 4 0 5 0 3 3 1 0 2

8. For the data below, make a frequency table and then draw a pie graph:

**Rolling a die**

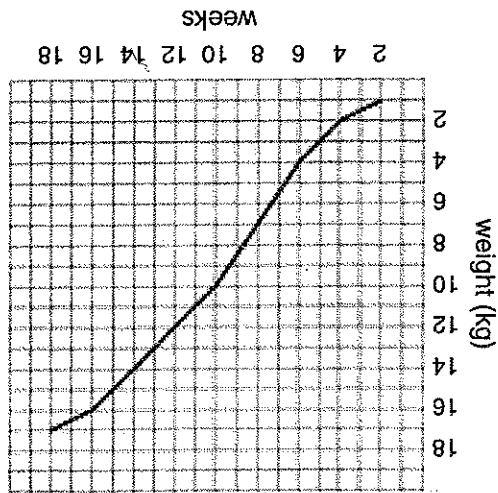
5 1 2 4 1 4 4 3 4 1  
 6 2 2 5 2 6 1 5 6 3  
 3 2 1 2 5 4 1 3 1 4  
 5 4 6 6 4 3 2 1 5 3

- (a) How many touchdowns did each team member score?  
 (b) How could you draw 1 touchdown, 5 touchdowns and 13 touchdowns?



Each football represents 4 touchdowns  
 season by four people.

10. The graph below shows how many touchdowns were scored in a



9. Dog

- (a) How much did the dog weigh at 17 weeks?  
 (b) What was the dog's weight gain between 2 and 4 weeks? 4 and 8 weeks? During which 5 week period did the weight of the dog increase by 50%?



## Grouped Data

Here are the results obtained by 50 Year 9 students in the end of year Maths exam:

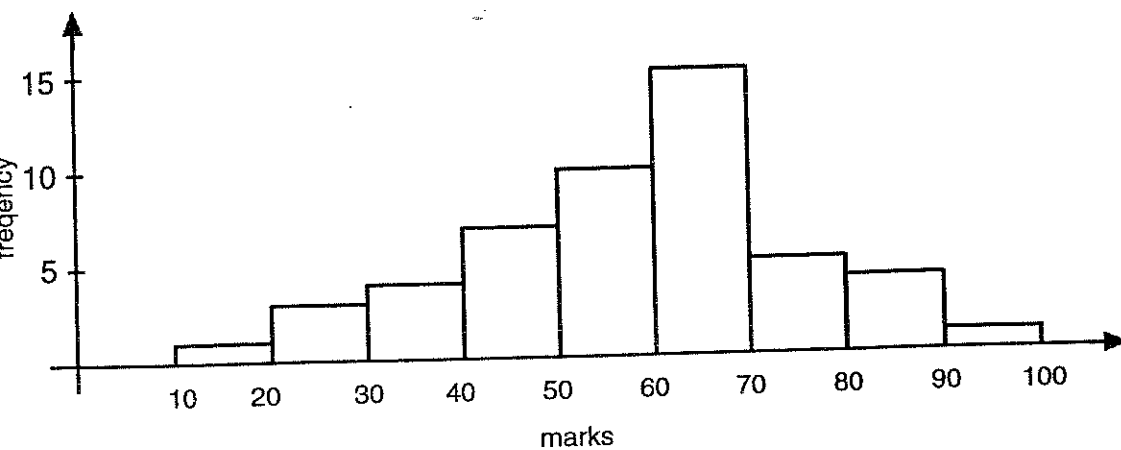
60 51 89 69 33 66 43 98 60 58  
 80 12 45 64 63 38 87 59 29 68  
 55 61 77 52 49 63 22 67 50 83  
 37 40 69 25 65 52 64 71 44 31  
 41 76 58 62 56 45 73 56 75 65

Because the marks are spread over a wide range, our frequency table will be different from the examples before.

In this case, we will put the data into **groups**.

Mark (in groups)	Tally	Frequency
0 - 9		0
10 - 19		1
20 - 29		3
30 - 39		4
40 - 49		7
50 - 59		10
60 - 69		15
70 - 79		5
80 - 89		4
90 - 100		1
		50

Graphing this .....



**Maths Exam Results**

$$\begin{aligned} \text{average} &= \frac{7+6+9+5+3+1+9+10}{8} \\ &= \frac{50}{8} \\ &= 6.25 \end{aligned}$$

**Example** Find the average (mean) of 7, 6, 9, 5, 3, 1, 9, 10

total number of results.

To calculate the mean (or average), add all the results and divide by the

**The Mean** Sometimes called the **Arithmetic Mean** or just the **average**.

There are three types of average.

usual or common.

The "average" is a common statistic which is calculated to show what is

## Averages

- Find the height of the tallest boy.
- Find the height of the smallest boy.
- Draw up a frequency table. Group the data in 4 cm groups, starting with the height of the shortest student.
- Draw a pictograph using  $\frac{1}{2}$  = 4 boys.
- Draw a pie graph.
- Draw a bar graph (histogram).

162	165	158	171	169	163	162	165
154	170	158	155	154	152	160	160
154	172	162	170	164	170	151	162
159	157	159	167	160	159	155	172
173	166	158	156	175	155	165	159
158	170	160	154	153	155	159	159
155	163						

1. The heights (in cm) of 50 Year 9 boys are:

### Exercise 3.2



**The Median** The median is the middle number.

To find the median, you must first write all the numbers from smallest to largest. Then find the middle result.

**Example 1** Find the median of

5, 7, 3, 6, 9, 10, 4, 7, 9

Write in order: 3, 4, 5, 6, 7, 7, 9, 9, 10

↑  
middle number

median = 7

**Example 2** Find the median of

1, 5, 3, 7, 10, 2

Write in order: 1, 2, 3, 5, 7, 10

↑  
middle

∴ median occurs between 3 and 5

median = 4

**The Mode** The mode is the result that occurs the most frequently. It is the most common result.

**Example** Find the mode of

4, 3, 1, 7, 9, 8, 4, 3, 1, 4

We can see that 4 is the most common result

∴ mode = 4

**Example 1** Find the mean, median and mode of

10, 30, 100, 40, 100, 50

$$\text{mean} = \frac{10 + 30 + 100 + 40 + 100 + 50}{6}$$

$$= \frac{330}{6}$$

$$= 55$$

median: 10, 30, 40, 50, 100, 100

↑  
middle

∴ median = 45

mode = 100

1. Find the mean, median and mode of the following data:
- |     |  |
|-----|--|
| (a) | 1, 3, 3, 4, 5, 7, 8                    |
| (b) | 6, 2, 2, 4, 5, 8, 2, 4, 1              |
| (c) | 11, 13, 15, 17, 17, 19                 |
| (d) | 38, 30, 20, 5, 27, 35, 8, 30           |
| (e) | 1, 6, 2, 5, 5, 6, 6, 4, 4, 3, 2, 1     |
| (f) | 22, 24, 24, 26, 27, 28, 30             |
| (g) | 19, 15, 9, 1, 19, 31, 18, 6            |
| (h) | 13, 13, 13, 13, 14, 14, 15, 15, 16, 28 |
| (i) | 34, 34, 35, 36, 36, 51                 |
| (j) | 1, 2, 3, 4, 5, 6, 7, 8, 9, 10          |
2. A cricketer has scores of 4, 29, 40, 0 and 29. Calculate the score they need in the next game for an average of 40.
3. A basketballer has scores of 14, 6, 18, 24, and 23. Calculate the number of points which need to be scored in the next game for an average of 20.
4. A class obtains the following marks for a test: 19, 19, 17, 17, 17, 16, 15, 15, 15, 14, 14, 13, 13, 12, 12, 11, 10, 9, 8.
- (a) Calculate the mean, median and mode.  
 (b) If two students had been absent, and obtained 4 and 5 for the test the next day, what would the mean, median and mode be now?

**Exercise 3.3**

mean =  $\frac{1+2+3+4+3+4+4+5+1}{9}$   
 $= \frac{27}{9}$   
 $= 3$   
 median: 1, 1, 2, 3, 3, 4, 4, 4, 5  
 ↓  
 middle  
 $\therefore$  median = 3  
 mode = 4

**Example 2**

Find the mean, median and mode of 1, 2, 3, 4, 3, 4, 4, 5, 1

- (c) Which statistic changed the least and which changed the most when these two extra results were included?
5. A fitness class runs the following distances in 2 minutes:  
520, 510, 500, 490, 480, 470, 470, 470, 470, 460, 460, 450, 440
- (a) Calculate the mean, median and mode.
- (b) If the two runners who ran 460 m ran again later and managed 590 m and 600 m, what would the mean, median and mode be if their new distances were substituted?
- (c) Which statistic changed least and which changed the most?
6. The average weight of two students is 46 kg. If one weights 42.5 kg, what is the weight of the other?
7. A class of 25 students had a median mark of 58 in English. How many students had a mark less than 58?
8. If the average weight of 8 men is 84 kg, what is the total weight of the 8 men?
9. In a netball team of 7 players, 3 are 150 cm tall, 2 are 145 cm tall, 1 is 182 cm tall and 1 is 165 cm tall.
- (a) What is the average height of the team?
- (b) What is the median height?
- (c) What is the modal height?

## Probability

The probability of something happening is a measure of how likely it is.

### Certain and impossible events

The probability of something happening which is impossible is equal to zero.

e.g. The probability that it will snow in Luganville is zero since it is an impossible event.

We would write this as  $\text{Pr}(\text{snow in Luganville}) = 0$

The probability of something happening which is **certain** is equal to **one**.

e.g. The probability that a student will be given Maths homework sometime this term is one since this is a certain event.

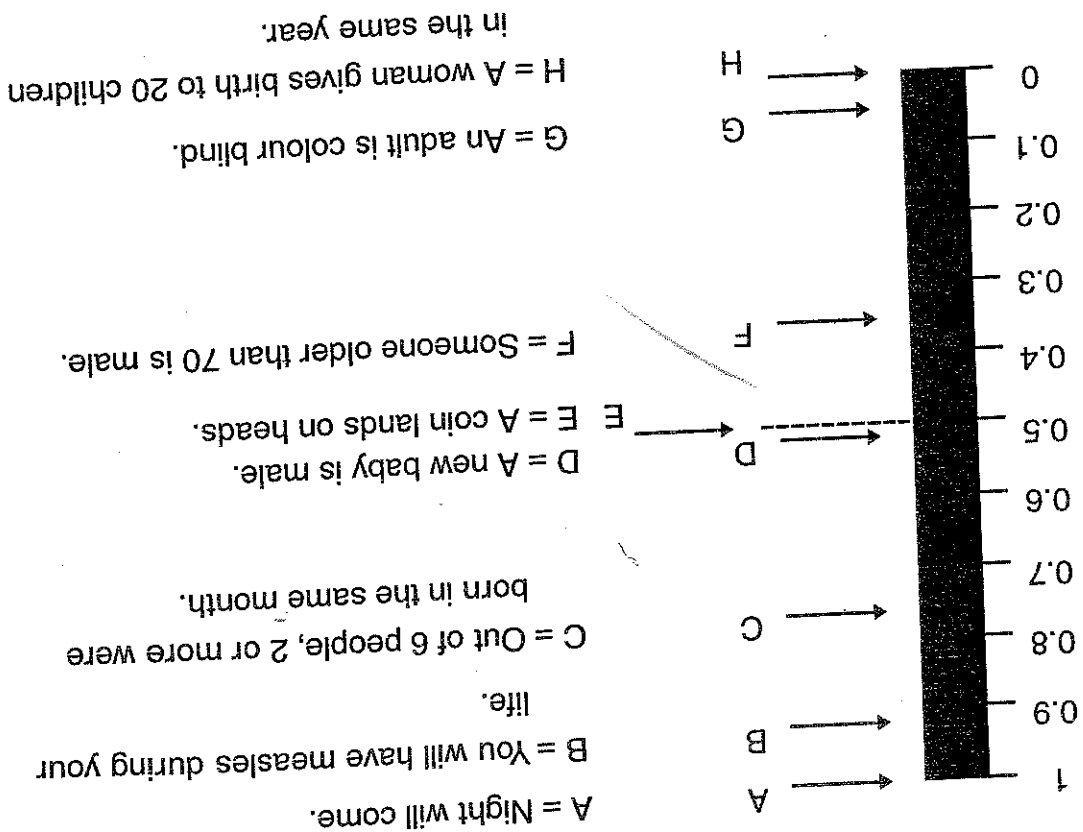
$\text{Pr}(\text{Maths homework this term}) = 1$

All probabilities are in between these two values.

Probabilities are written as a fraction or decimal.

### Exercise 3.4

The probabilities of some events are shown on this scale:



Do you agree with these probabilities?

1. Draw a scale from 0 to 1 and mark the position of these events:

- (a) A ni-Vanuatu teenager has black hair.
  - (b) A baby will be left-handed.
  - (c) Two people in your class have the same first name.
  - (d) You will marry someone with blond hair.
  - (e) You will be involved in a car accident during your life.
  - (f) You will travel overseas within the next 10 years.
  - (g) There will be a cyclone in July.
2. Draw a scale and mark the position of the following events.
- (a) It will rain at your school tomorrow.
  - (b) The most popular music in your family is reggae.
  - (c) The price of rice will rise this year.
  - (d) Unemployment will rise this year.

- (e) There will be a plane crash in Vanuatu this week.  
 (f) Fewer tourists will come to Vanuatu next year.
3. (a) Write down four events which are certain to occur.  
 (b) Write down four events which are impossible.

## Calculating Probability

If we want to calculate the probability of an event or outcome, we need to know how many outcomes there are in total and how many outcomes were successful (this means how many were the ones we wanted).

$$\text{Pr (success)} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes.}}$$

Let us look at some examples:

**Example 1** A boy own 10 cassette tapes. 6 are reggae tapes and 4 are rock music. If he chooses one tape without looking at it, what is the probability that

(a) it is a reggae tape

$$\begin{aligned} \text{Pr (reggae)} &= \frac{\text{number of reggae tapes}}{\text{total number of tapes}} \\ &= \frac{6}{10} \\ &= \frac{3}{5} \end{aligned}$$



Always cancel fractions if possible.

(b) it is a rock music tape

$$\begin{aligned} \text{Pr (rock)} &= \frac{\text{number of rocktapes}}{\text{total number of tapes}} \\ &= \frac{4}{10} \\ &= \frac{2}{5} \end{aligned}$$

**Example 2**

A sports bag contains 6 soccer balls, 12 tennis balls and 2 rugby balls. A ball falls out of the bag. What is the probability it was

(a) a soccer ball  $\Pr(\text{soccer ball}) = \frac{20}{6} = \frac{10}{3}$

(b) a tennis ball  $\Pr(\text{tennis ball}) = \frac{20}{12} = \frac{5}{3}$

(c) a rugby ball  $\Pr(\text{rugby ball}) = \frac{20}{2} = \frac{10}{1}$

**Exercise 3.5**

1. A six-sided dice is rolled.

- (a) List all the possible outcomes.  
 (b) What are the probabilities of the top number being:

(i) a 5?

(ii) an even number?

(iii) an odd number?

(iv) a number less than 5?

(v) a number less than 7?

(vi) a number bigger than 6?

2. Ten cards are numbered from 1 up to 10.

(a) What is the probability of choosing number 10?

(b) List all the even numbers. What is the probability of choosing an even number?

(c) List all the prime numbers. What is the probability of choosing a prime number?

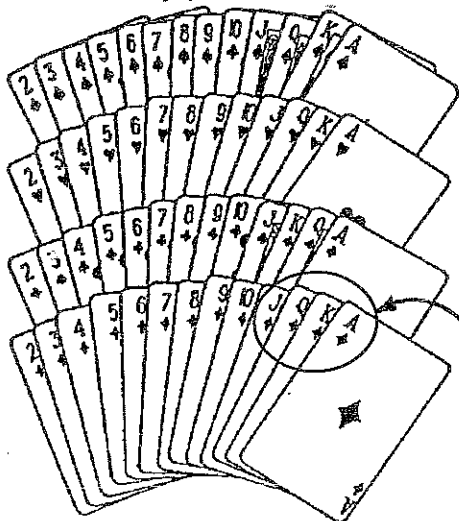
(d) What is the probability of choosing a number less than 4?

(e) Calculate  $\Pr(6)$

(f) Calculate  $\Pr(\text{not } 6)$

3. If the weather bureau said "The probability that it will rain tomorrow is  $\frac{1}{5}$  or 0.2", what is the probability it will not rain?
4. There are 10 library books on the shelf. 6 are green and 4 are red. Two of the green ones are maths books. If a book is chosen, what is the probability that it is:
- |                      |                                |
|----------------------|--------------------------------|
| (a) a green book     | (e) either red or green        |
| (b) a red book       | (f) a green maths book         |
| (c) a blue book      | (g) green but not a maths book |
| (d) not a green book | (h) not a maths book           |
5. In the Year 9 class there are 12 girls and 15 boys. Four of the girls have names beginning with T and three begin with M. Two of the boys' names begin with T and three begin with M. Answer the following questions:
- (a) What is the probability of choosing a girl? Pr (g)
- (b) What is the probability of choosing a boy? Pr (b)
- (c) What is the probability of choosing a girl with a name starting with T? Pr (gT)
- (d) What does Pr (gM) mean?
- (e) Calculate Pr (gM)
- (f) Calculate Pr (bT)
- (g) Calculate Pr (M)
- (h) Calculate Pr (T)
- (i) Calculate Pr (not M)
- (j) Calculate Pr (not T)

6. An ordinary pack of playing cards looks like this:



There are 52 cards.

13 hearts ♥ which are red

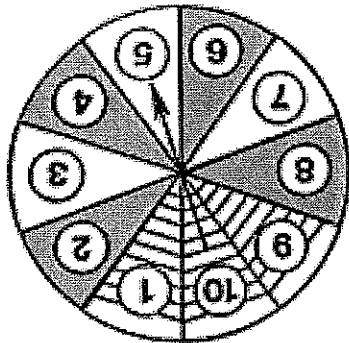
13 diamonds ♦ which are red

13 spades ♠ which are black

13 clubs ♣ which are black.

Picture cards are the Jack, Queen, King and Ace.

	$\frac{4}{10}$	$\frac{1}{3}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{4}{3}$	$\frac{1}{10}$	$\frac{5}{4}$	$\frac{1}{3}$	$\frac{3}{10}$	$\frac{1}{7}$	$\frac{5}{12}$	$\frac{3}{5}$	$\frac{12}{5}$	$\frac{2}{1}$	$\frac{10}{9}$
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probability that it will stop on a region that is:

(H) striped?  
 (L) either white or shaded?  
 (A) either striped or numbered 7?  
 (Y) either numbered 6 or numbered 3?  
 (S) numbered 4?  
 (H) not numbered 4?  
 (F) not shaded?  
 (H) either striped or shaded or numbered 7?

A spinner is shown at the right. If the arrow is spun, what is the

(R) either numbered 3 or numbered 9?

(S) either white or numbered 5?

(E) either white or striped?

(L) either striped or shaded?

(E) numbered 10?

(I) white?

(T) shaded?

(E) striped?

the right. What is the probability that the card is:

Suppose that a card is drawn at random from the 12 cards shown at

Keep working and you will discover the answer to the title question.

Find the answer to any question below in the boxes at the bottom of the

page. Write the letter of that question in the box above its correct answer.

### Why Are Oysters Greedy?

- (i) a black ace
- (g) a picture card
- (e) a heart or a diamond
- (c) a red card
- (a) a spade
- (j) an even number
- (h) not a picture card
- (f) not a club
- (d) the king of hearts
- (b) a ten

it is:

If a card is chosen at random (without looking) what is the chance that



## Probability Experiments

### Example

I tossed a 20vt coin 40 times and obtained the following results:

H	T	H	T	H	H	H	T	H	T
H	T	T	H	T	H	H	T	T	H
H	T	H	T	T	T	T	T	T	T
T	H	T	H	T	T	T	H	T	T

H = Heads (symbol of Vanuatu)

T = Tails (coconut crab)

When I counted up my results, I made up a table:

H	T
16	24



From my results, I could say that, for our coin, the probability of getting a tail is:

$$\begin{aligned}\Pr(T) &= \frac{24}{40} \\ &= \frac{3}{5}\end{aligned}$$

However, I was expecting to get 20 tails and 20 heads. Does this mean my coin wasn't fair? No, it just means that I didn't toss the coin enough times. If I had tossed it 4000 times (very boring!) the results would be very close to 2000 heads and 2000 tails.

### Exercise 3.6

For each of the following experiments

- Predict the number of times each of the outcomes will occur.
  - Conduct the experiment and record the results.
  - Draw up a table of results.
  - Calculate the probabilities of each of the outcomes of the experiment.
  - Compare your results with your predictions.
- Toss a 10vt coin and a 20vt coin 60 times. Record how many times you get 2 heads, 1 head and 1 tail, 2 tails.
  - Drop a drawing pin onto your page 100 times. Record how many times you get point up  or point down .

Sometimes we need to look at the probability involving two or more different events.

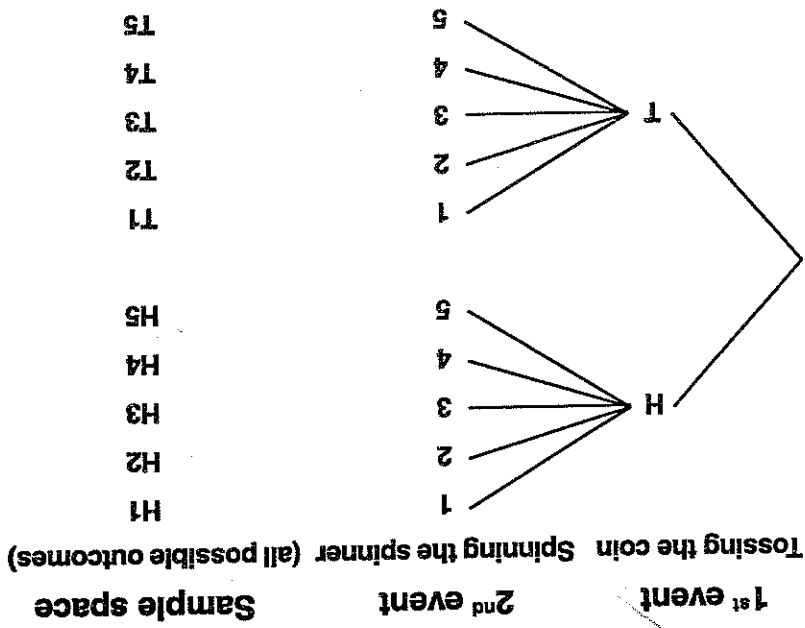
## Multiple Events

3. Choose a page out of a book you are reading. Record how many times each letter appears.
4. From a set point, throw a basketball at the goal 100 times. How many goals do you score? How many misses?

### Example 1

Tossing a coin and then spinning a spinner numbered 1 to 5.

The easiest way of investigating this to use a **tree diagram**.



There are 10 possible results.

We can now find the following probabilities.

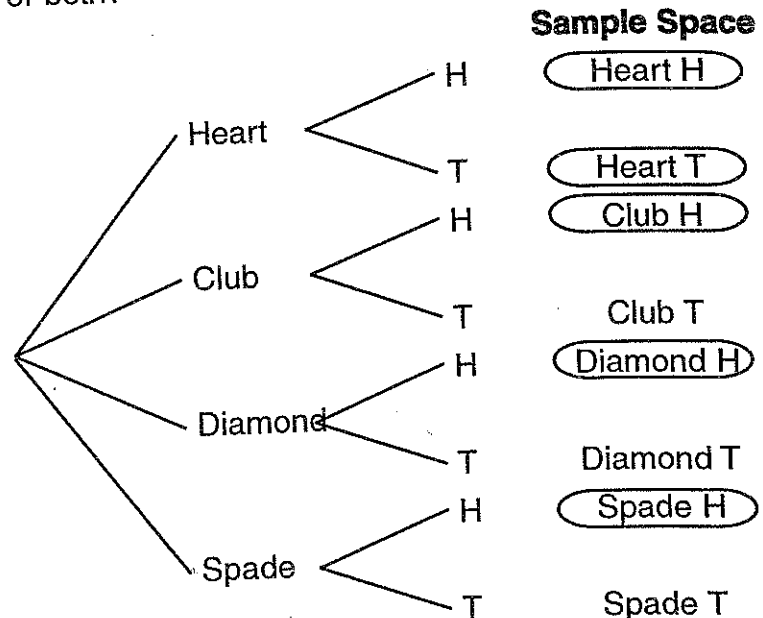
$$\text{Pr (H and even number)} = \frac{10}{2} = \frac{5}{1}$$

$$\text{Pr (H and even number)} = \frac{10}{4} = \frac{5}{2}$$

$$\text{Pr (even number or T)} = \frac{10}{7}$$

**Example 2**

When choosing a card from a pack and then tossing a coin, what is the probability of getting a heart or a head or both?



5 of the results contain a heart, a head or both.

$$\Pr(\text{Heart or head or both}) = \frac{5}{8}$$

**Exercise 3.7**

Draw tree diagrams to help you answer the following:

1. When tossing a coin and then spinning a spinner numbered 1 to 4, what is the probability of getting a tail or a 4 or both?
2. If a card is drawn from a pack and then a die is rolled, what is the probability of
  - (a) a club or a six or both?
  - (b) a red card or a number under 5?
3. Three coins are tossed. What is the probability of getting 3 heads or only one head?
4. There are four girls – Mere, Susan, Crystal and Unes – and four boys – Kalo, Tom, Jimmy and Webster – in the same class. Their teacher wants to choose one girl and one boy to form a committee. Find the probability that the committee contains:

- (a) Susan and Crystal
- (b) Susan or Crystal
- (c) Webster
- (d) Not Kalo

5. Justin is choosing his subjects for Year 11. He must choose one subject from each block.

Block 1	French	Computers	Chemistry
Block 2	Physics	Geography	
Block 3	History	Biology	

- (a) List all the possible subject combinations that Justin can choose.
- (b) Find the following:
  - (i) Pr (Computers, Physics, Biology)
  - (ii) Pr (2 science subjects)
  - (iii) Pr (not computers)
  - (iv) Pr (History or Geography)

6. At a birthday feast, Jacklyn can have chicken, beef or fish and then manioc, taro or yam. For sweets she can choose cake or ice-cream. If she can only choose one of each type of food, find:

- (a) Pr (beef or fish)
- (b) Pr (not ice-cream)
- (c) Pr (chicken, rice and cake)
- (d) Pr (beef and yam, with either sweet)

**Exercise 3.8 Mixed Problems**

1. Estimate the following probabilities:
  - (a) A student will get homework next Monday.
  - (b) The price of tinned fish will fall next week.
  - (c) It will not rain tomorrow.
  - (d) A plane will crash sometimes this year.
2. List 2 events that have zero probability and 2 events that have a probability of 1.
3. A 6 sided die is tossed. Find the probabilities of throwing:
  - (a) a six
  - (b) an even number
  - (c) a 6 or a 1
  - (d) a five or a multiple of 3
  - (e) a prime number

4. Find the probability of choosing from a pack of cards.
- (a) an ace
  - (b) a club
  - (c) a king or a club
  - (d) a king or a two
  - (e) a black even number
  - (f) not a jack or a heart
5. A bag contains 10 white stones, 6 black stones and 8 grey stones. Find the probability of choosing:
- (a) a white Stone
  - (b) a black stone
  - (c) a black or a grey stone
  - (d) not a grey stone
6. Michelle is about to have a baby. It could be a boy or a girl. It could be born on Monday, Tuesday or Wednesday. Draw a tree diagram to show this information.
- (a) List the sample space.
  - (b) What is the probability of a boy born on Tuesday?
7. (a) Draw a tree diagram to show the results when a die is rolled and a coin is tossed.
- (b) Find Pr (a six or a head or both).
  - (c) Find Pr ( a number over two and a tail).
8. (a) A coin is tossed 40 times. How many tails would you expect?
- (b) A die is tossed 30 times. How many 4s would you expect?
  - (c) 20 cards are chosen from a pack. How many hearts would you expect?
9. Two dice are tossed.
- (a) Write out the sample space of all the different combinations.
  - (b) What is the probability of throwing a pair (two numbers the same)?
  - (c) What is the probability of throwing a total of 10 or more?
  - (d) What is the probability of throwing a total of 6 or 7?
  - (e) A game is invented where a person will win if they throw a total which is a multiple of 3. What is the chance of winning the game?