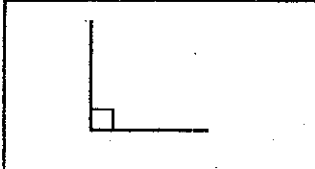

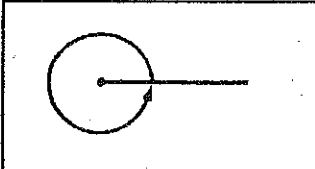
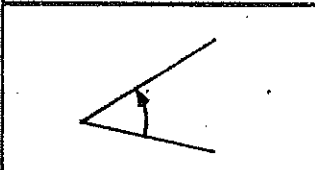
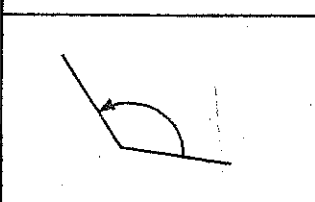
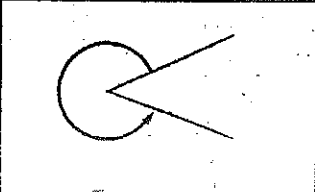
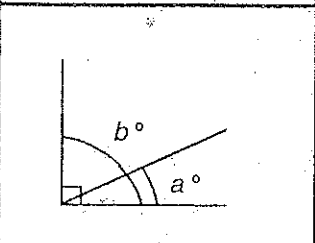
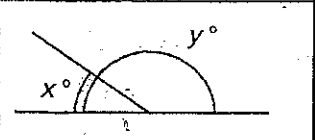


Topic Three: Angles and Compass Constructions

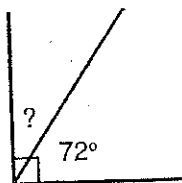
Revision of Angle Facts

	Right angle	90°
	Straight angle	180°
	One revolution	360°
	Acute angle	0° angle 90°
	Obtuse angle	90° angle 180°
	Reflex angle	180° angle $\neq 360^\circ$
	Complementary angles	$a^\circ + b^\circ = 90^\circ$
	Supplementary angles	$x^\circ + y^\circ = 180^\circ$

Example 1 Name the type of angle which has a measure of:

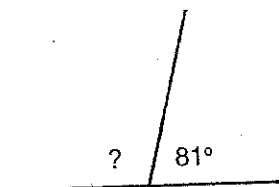
- (a) 173° - obtuse angle
- (b) 200° - reflex angle
- (c) 3° - acute angle

Example 2 Give the complement of 72° .



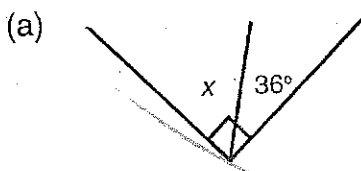
$$\begin{aligned}\text{Complement of } 72^\circ &= 90^\circ - 72^\circ \\ &= 18^\circ\end{aligned}$$

Example 3 Give the supplement of 81° .

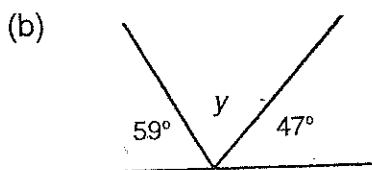


$$\begin{aligned}\text{Supplement of } 81^\circ &= 180^\circ - 81^\circ \\ &= 99^\circ\end{aligned}$$

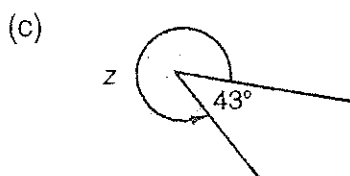
Example 4 Find the value of the pronumeral in the following:



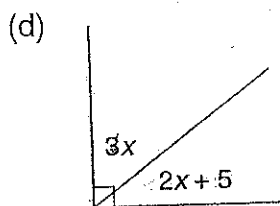
$$\begin{aligned}x &= 90 - 36 \\ &= 54^\circ\end{aligned}$$



$$\begin{aligned}y &= 180 - 59 - 47 \\ &= 74^\circ\end{aligned}$$



$$\begin{aligned}z &= 360 - 43 \\ &= 317^\circ\end{aligned}$$



$$\begin{aligned}3x + 2x + 5 &= 90 \\ 5x + 5 &= 90 \\ -5 &\quad -5 \\ 5x &= 85 \\ x &= 17^\circ\end{aligned}$$

Exercise 3.1

1. Name the type of angle which has a measure of:

- | | |
|-----------------|-----------------|
| (a) 132° | (f) 33° |
| (b) 163° | (g) 92° |
| (c) 49° | (h) 337° |
| (d) 205° | (i) 301° |
| (e) 253° | (j) 109° |

2. Give the complement of:

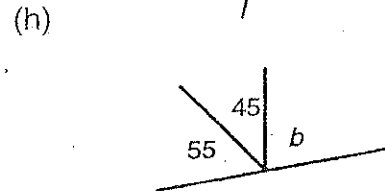
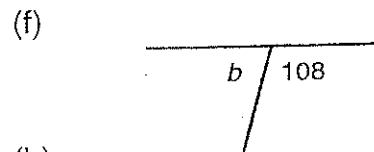
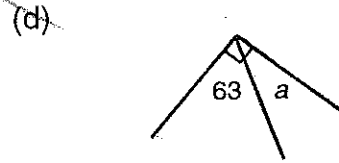
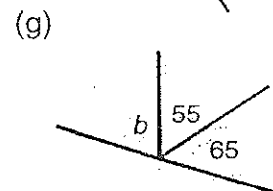
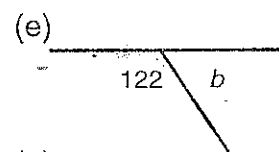
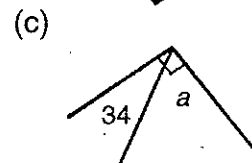
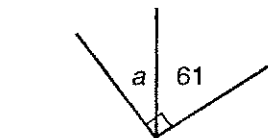
- | | |
|----------------|----------------|
| (a) 20° | (e) 42° |
| (b) 41° | (f) 56° |
| (c) 68° | (g) 88° |
| (d) 87° | (h) 71° |

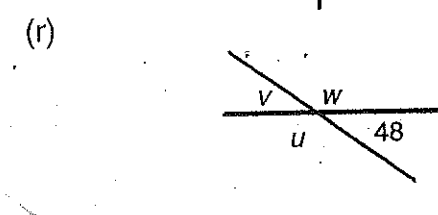
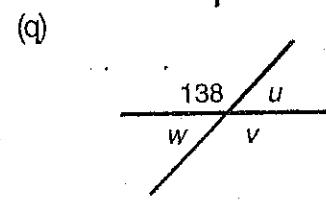
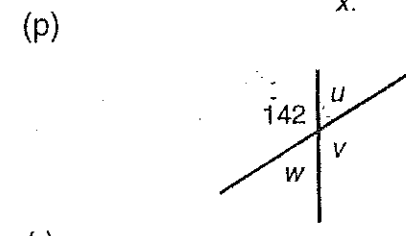
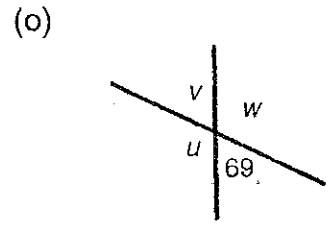
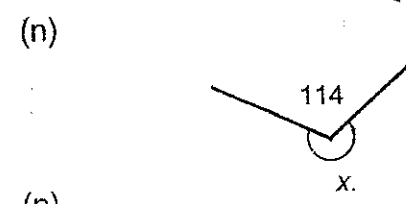
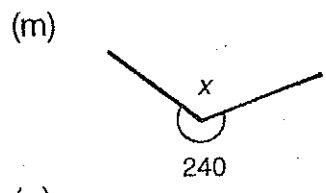
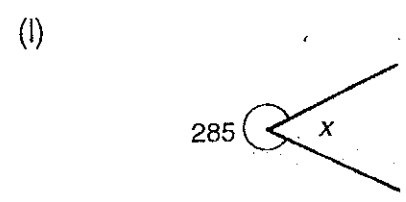
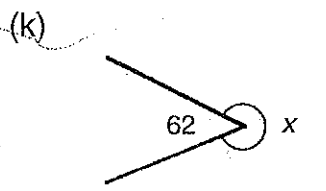
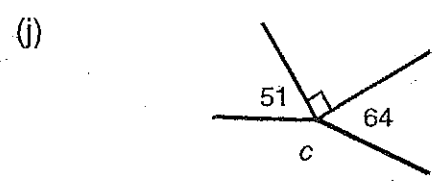
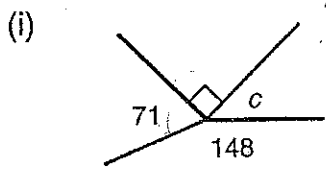
3. Give the supplement of:

- | | |
|----------------|-----------------|
| (a) 19° | (e) 110° |
| (b) 39° | (f) 122° |
| (c) 65° | (g) 173° |
| (d) 88° | (h) 169° |

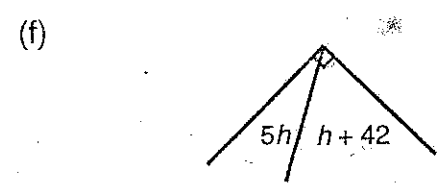
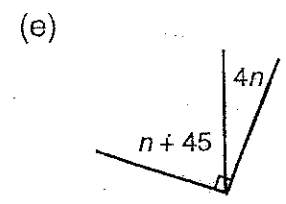
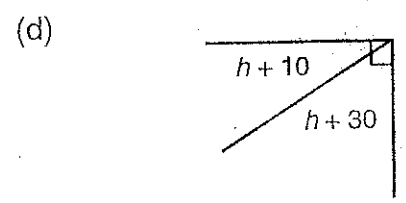
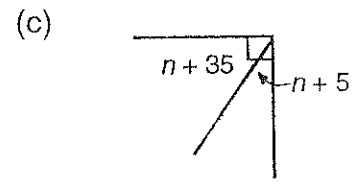
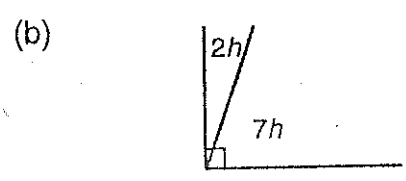
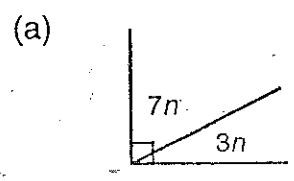
4. Find the value of the pronumerals:

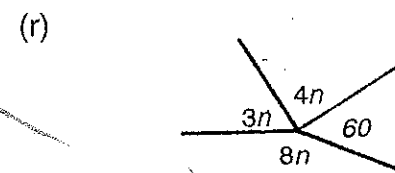
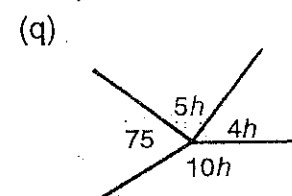
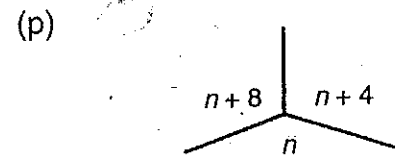
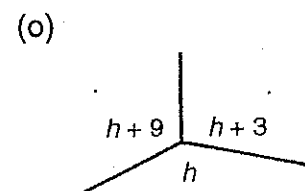
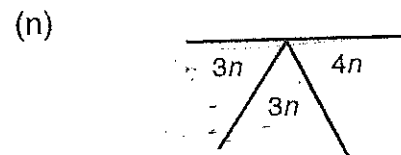
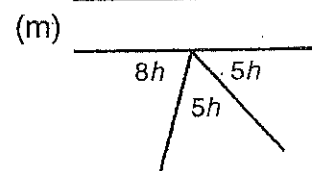
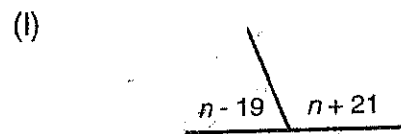
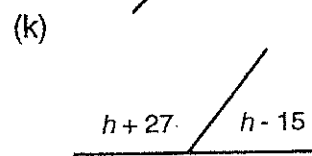
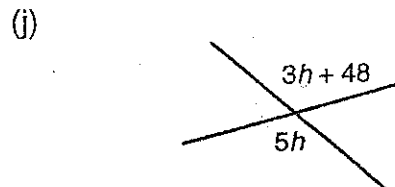
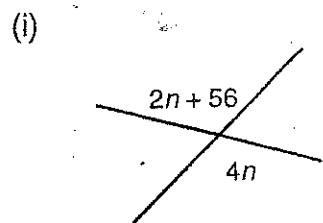
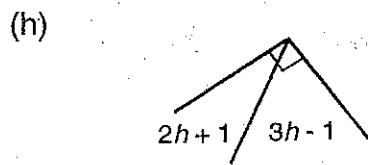
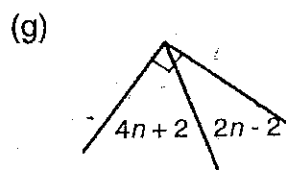
- (a) (b)





5. Use equations to find the value of the pronumeral:





6. Through what angle does the hour hand of a clock turn in:

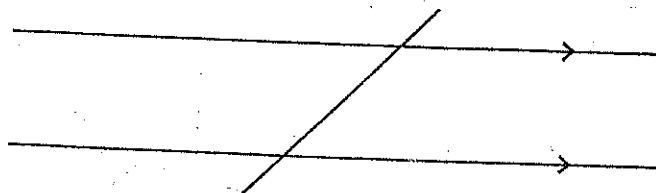
- (a) 4 hours (d) 6 hours
 (b) 1 hour (e) 30 hours
 (c) 15 hours (f) 21 hours

7. Through what angle does the minute hand of a clock turn in:

- (a) 15 minutes (c) 1 minute
 (b) 50 minutes (d) 8 minutes

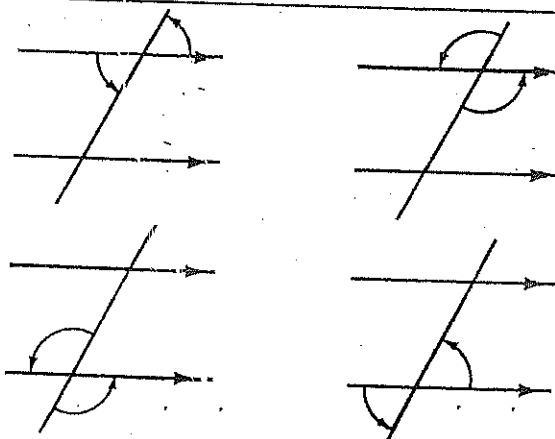
Angles in Parallel Lines

When a pair of parallel lines are crossed by another line, eight angles are formed.

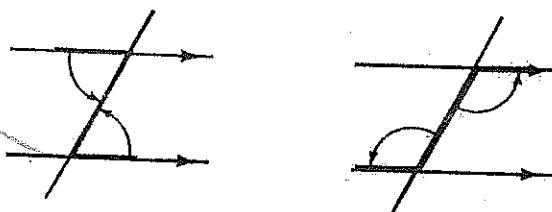


Angle facts

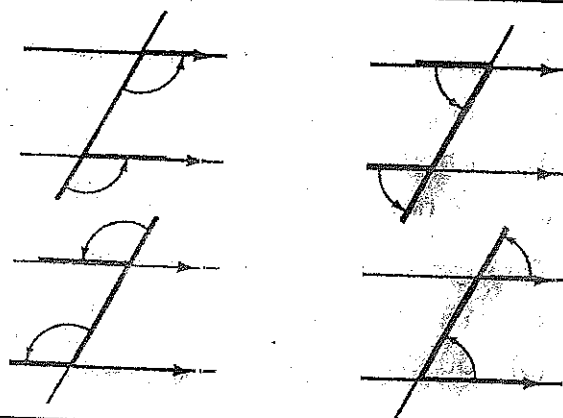
1. Pairs of vertically opposite angles are equal.



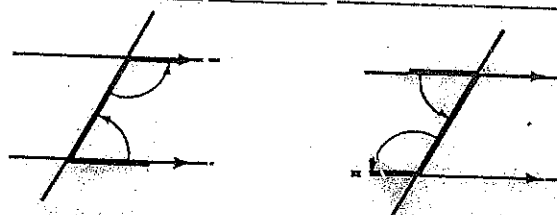
2. Pairs of alternate angles are equal. (Angles form a 'Z'.)



3. Pairs of corresponding angles are equal. (Angles form an 'F')



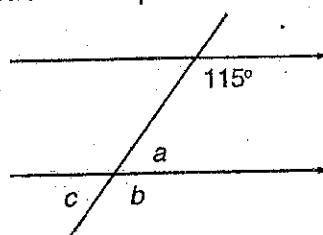
4. Pairs of allied (cointerior) angles are supplementary. (Angles form a 'C').



Example

Find the value of the pronumerals:

(a)



$$115^\circ + a = 180^\circ$$

So $a = 180 - 115$
 $= 65^\circ$ (cointerior)

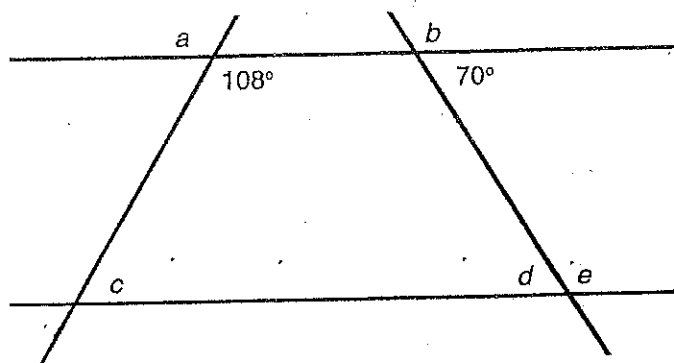
a and c are vertically opposite.

So $c = 65^\circ$

115° and b are corresponding.

So $b = 115^\circ$

(b)



$a = 108^\circ$ (vertically opposite)

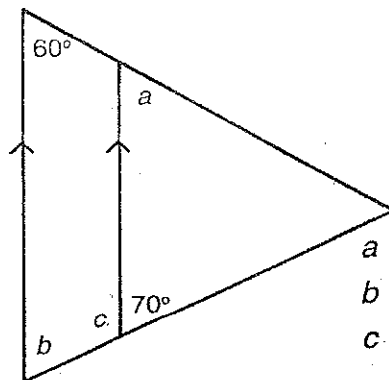
$b = 110^\circ$ (corresponding)

$c = 180 - 108$
 $= 72^\circ$ (cointerior)

$d = 70^\circ$ (alternate)

$e = -180 - 70$
 $= 110^\circ$ (cointerior)

(c)



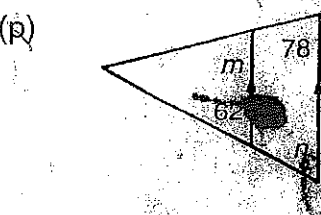
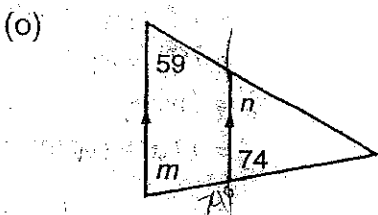
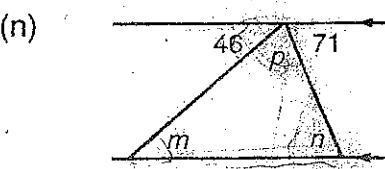
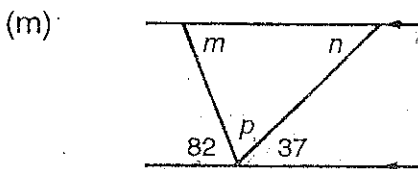
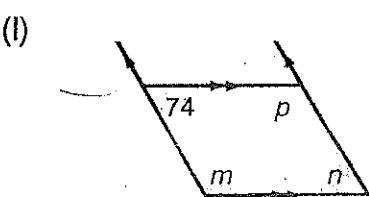
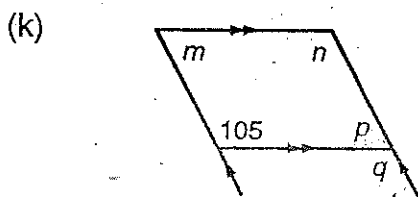
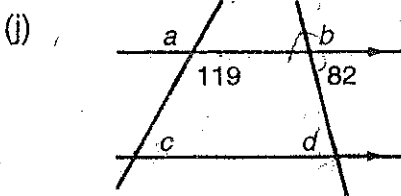
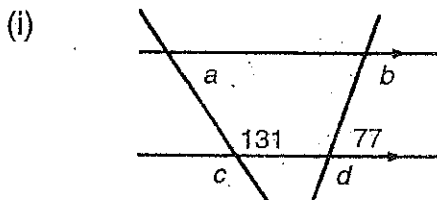
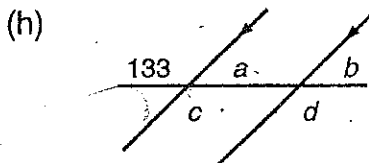
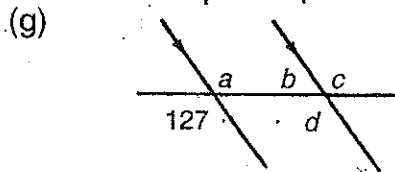
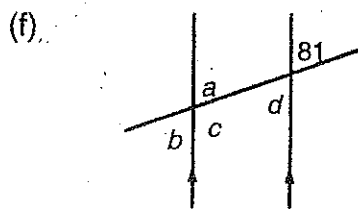
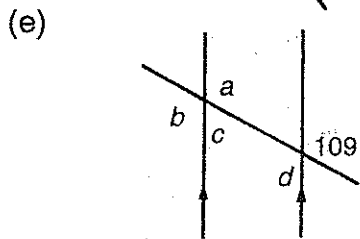
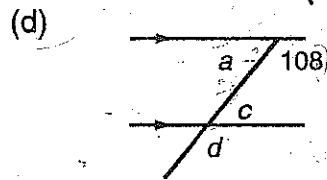
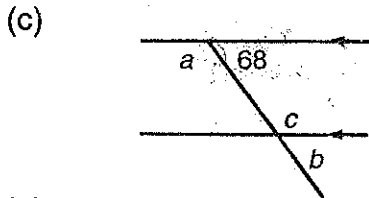
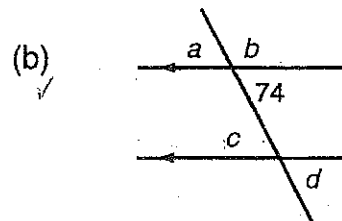
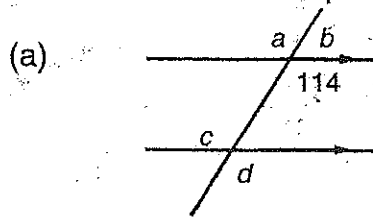
$a = 60^\circ$ (corresponding)

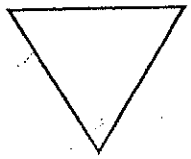
$b = 70^\circ$ (corresponding)

$c = 180 - 70$
 $= 110^\circ$ (cointerior)

Exercise 3.2

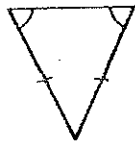
1. Find the values of the pronumerals:





Equilateral:

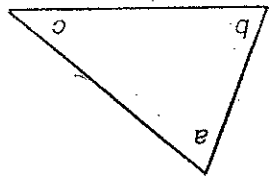
3 equal sides
all angles equal to 60°



Isosceles:

2 equal sides
2 angles equal

Two special triangles are:

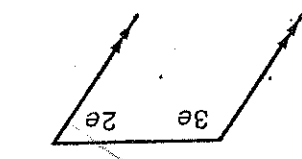


The sum of the angles inside a triangle = 180°
 $a + b + c = 180^\circ$

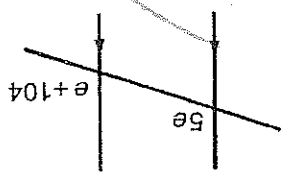
Facts you should know about triangles!

Angles and Triangles

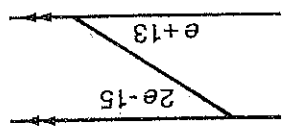
2. Use equations to find the values of the pronumerals:



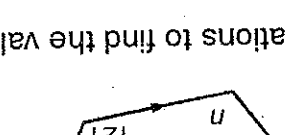
(a)



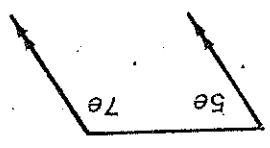
(b)



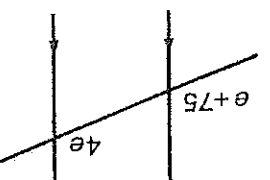
(c)



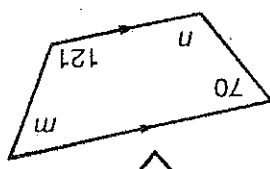
(d)



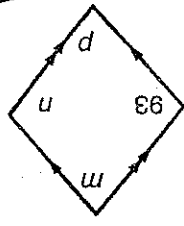
(e)



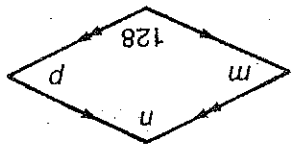
(f)



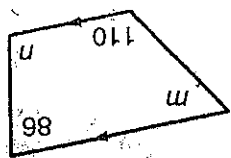
(a)



(b)



(c)

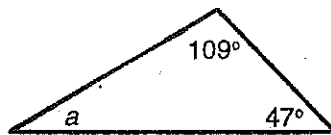


(d)

Examples

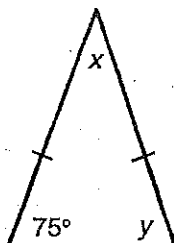
Find the value of the pronumerals:

(a)



$$a = 180^\circ - 109^\circ - 47^\circ = 24^\circ$$

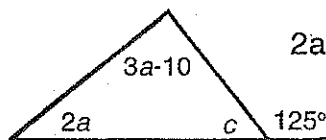
(b)



Isosceles triangle

$$\begin{aligned} \therefore y &= 75^\circ \\ x &= 180 - 75 - 75 \\ &= 30^\circ \end{aligned}$$

(c)



$$\begin{aligned} c &= 180 - 125^\circ \\ &= 55^\circ \\ 2a + 3a - 10 + 55 &= 180^\circ \\ 5a + 45 &= 180 \\ 5a &= 135 \\ a &= 27^\circ \end{aligned}$$

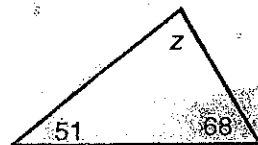
Exercise 3.3

1. Find the value of the pronumerals:

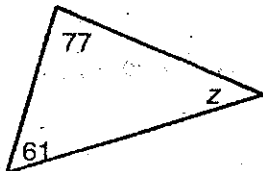
(a)



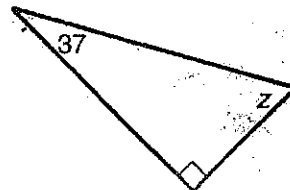
(b)



(c)



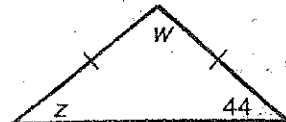
(d)



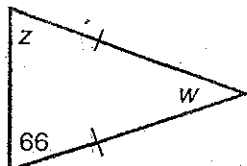
(e)



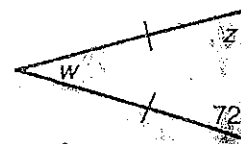
(f)



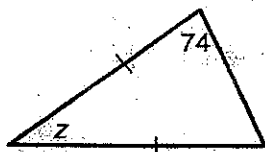
(g)



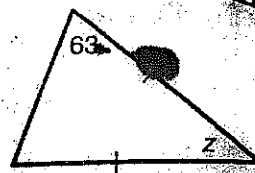
(h)



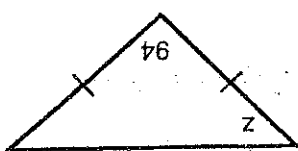
(i)



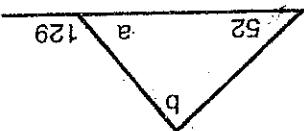
(j)



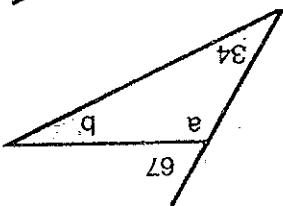
Use equations to find the pronumeral:



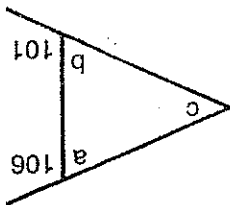
(i)



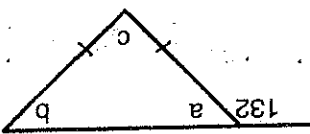
(n)



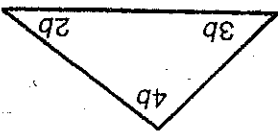
(p)



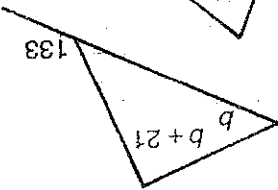
(r)



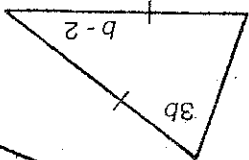
(t)



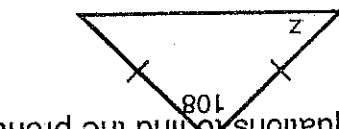
(b)



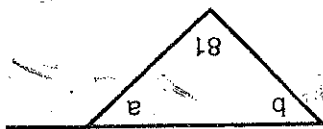
(d)



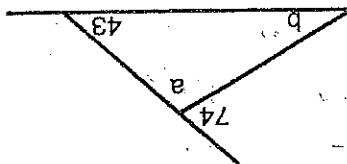
(f)



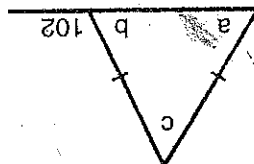
(m)



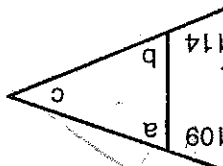
(o)



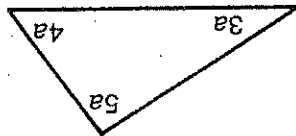
(q)



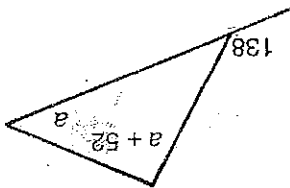
(s)



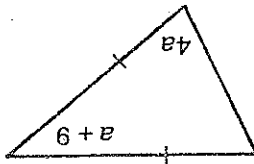
2. (a)



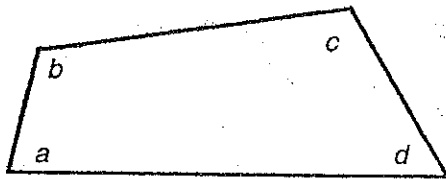
(c)



(e)



Angles and Quadrilaterals



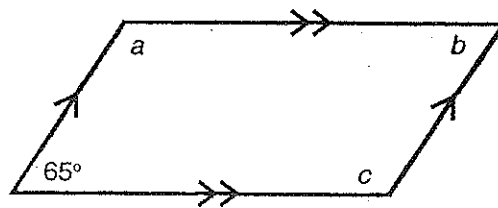
The angles inside any quadrilateral add up to 360° .

$$a + b + c + d = 360^\circ$$

Examples

Find the value of the pronumerals.

(a)



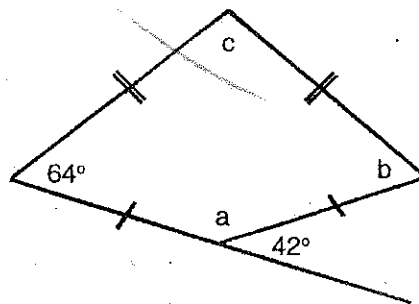
$$b = 65^\circ$$

$$a = 180 - 65$$

$$= 115^\circ$$

$$c = 115^\circ$$

(b)



$$a = 180 - 42$$

$$= 138^\circ$$

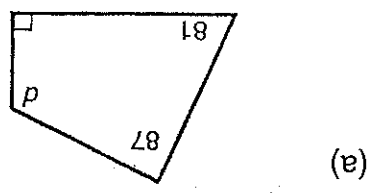
$$b = 64^\circ$$

$$c = 360 - 138 - 64 - 64$$

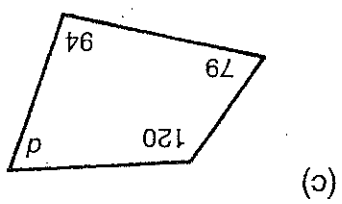
$$= 94^\circ$$

Exercises 3.4

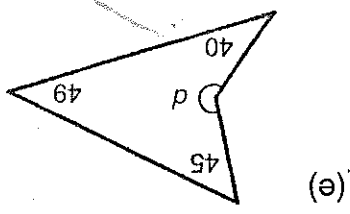
1. Find the value of the pronumerals:



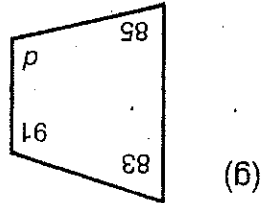
(a)



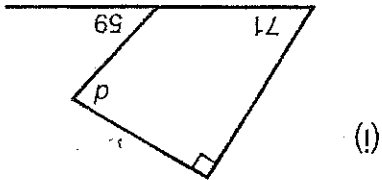
(c)



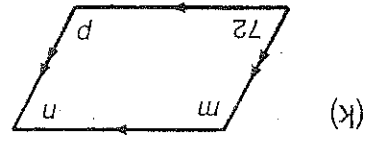
(e)



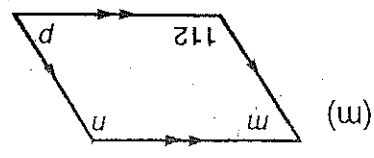
(g)



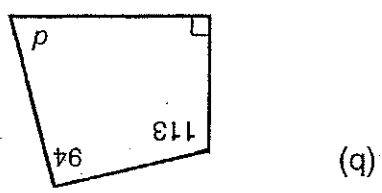
(i)



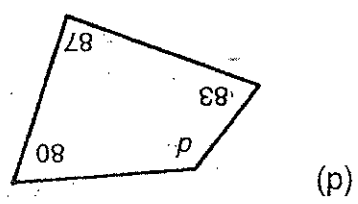
(k)



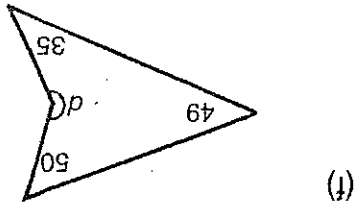
(m)



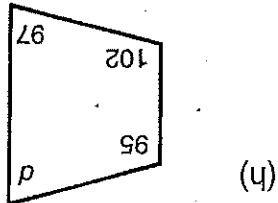
(b)



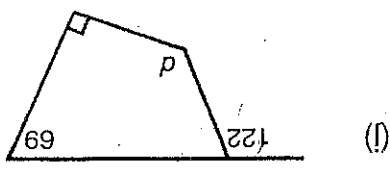
(d)



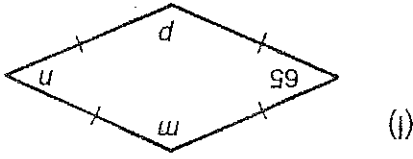
(f)



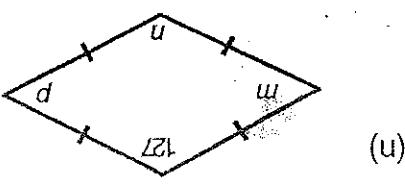
(h)



(j)

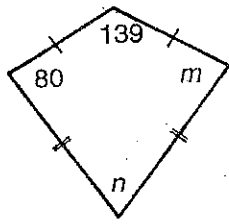


(l)

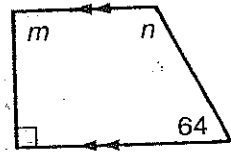


(n)

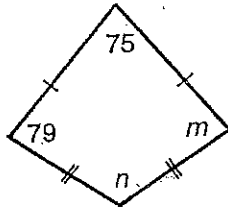
(o)



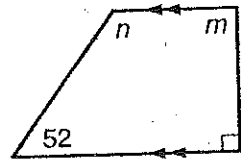
(p)



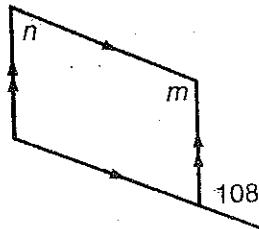
(q)



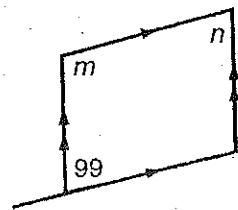
(r)



(s)

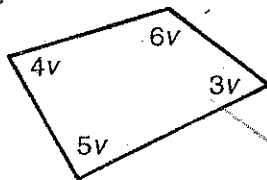


(t)

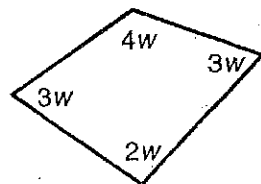


2. Use equations to find the value of the pronumerals:

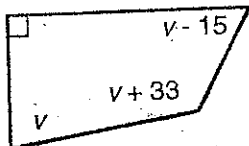
(a)



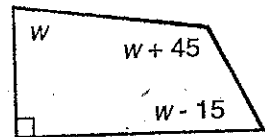
(b)



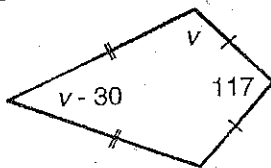
(c)



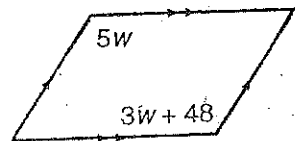
(d)



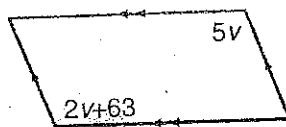
(e)



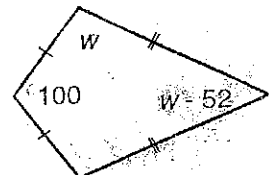
(f)



(g)




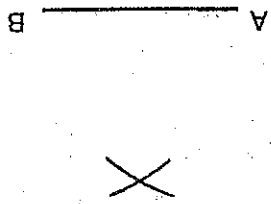

(h)



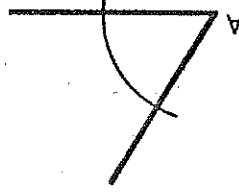
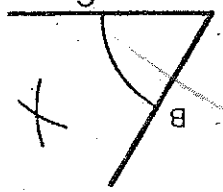

Compass Constructions

Below are six basic constructions:

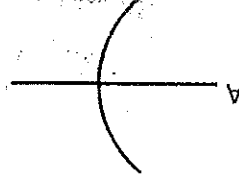
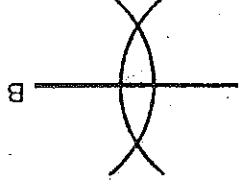

Angle of 60°

<p>Draw an interval.</p> 	<p>Draw arcs with radius AB and centres at A and B.</p> 	<p>Draw the angle.</p> 
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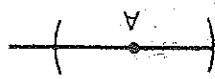
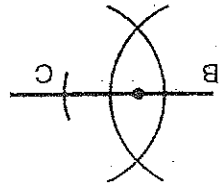

Bisector of an angle

<p>Draw an arc with centre at A.</p> 	<p>Draw arcs with centres at B and C.</p> 	<p>Draw the bisector.</p> 
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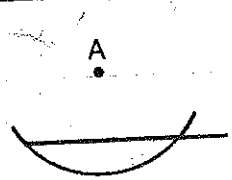
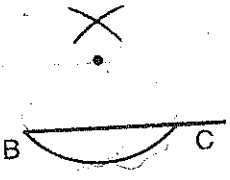
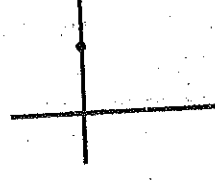
Perpendicular bisector of a line

<p>Draw an arc with centre A.</p> 	<p>Draw an arc with centre B.</p> 	<p>Draw the perpendicular bisector.</p> 
---	---	---

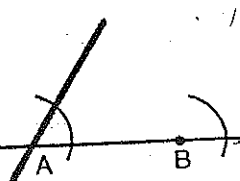
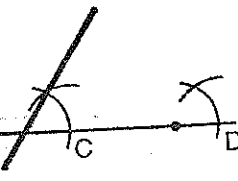
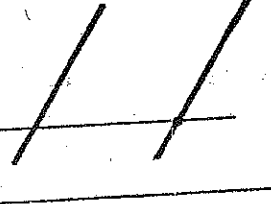
Perpendicular through a point on a line

<p>Draw arcs with centre A.</p> 	<p>Draw arcs with centres at B and C.</p> 	<p>Draw the perpendicular.</p> 
---	---	--

Perpendicular from a point to a line

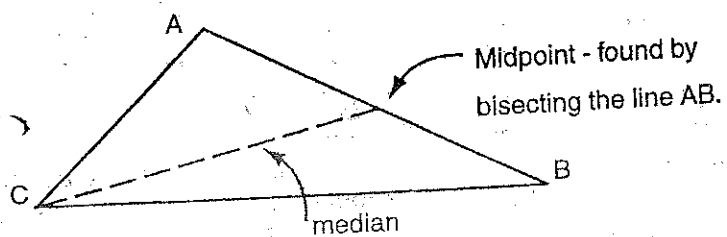
		
<p>Draw arcs with centre A.</p>	<p>Draw arcs with centres at B and C.</p>	<p>Draw the perpendicular.</p>

Parallel line through a point

		
<p>Draw a line from A through B. Draw arcs with centres at A and B.</p>	<p>Draw arcs with centres at C and D.</p>	<p>Draw the parallel line.</p>

Finding the Median

The line joining the point (or vertex) of a triangle to the midpoint of the opposite side is called the **median**.



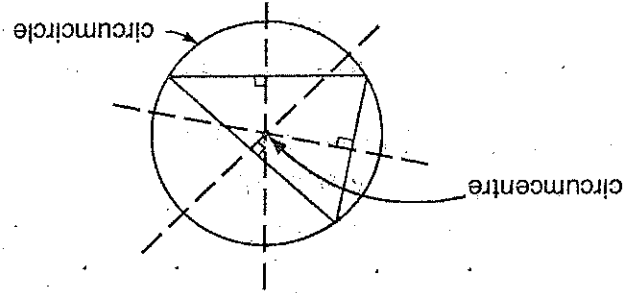
The point of intersection of the medians of a triangle is called the **centroid**.

1. (a) Rule a line 6 cm long.
 - (b) Draw an angle of 60° on the line.
 - (c) Continue to construct an equilateral triangle.
2. Construct a triangle with sides of 5 cm, 6 cm and 7.5 cm.
 3. Using your protractor, draw an angle of 80° . Now bisect the angle. Check your result with the protractor.

For all these questions you will need a compass, a ruler and a sharp pencil.

Exercise 3.5

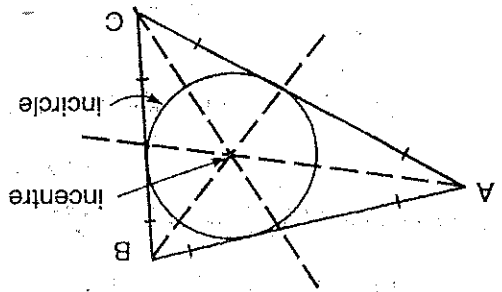
The circumcentre is the centre of the **circumcircle**, the circle which passes through all 3 vertices of the triangle.



The point of intersection of the perpendicular bisectors of the sides of a triangle is called the **circumcentre**.

Circumcentre and Circumcircle

The incentre is the centre of the **incircle**, the circle which touches all three sides of the triangle.



The point of intersection of the angle bisectors of a triangle is called the **incentre**.

Incentre and Incircle

4. Draw a line 11 cm long. Construct its perpendicular bisector.
5. Draw an angle of 142° using a protractor. Bisect the angle.
6. Draw a line 12 cm long. Mark a point, P, approximately 5 cm above the line. Draw a perpendicular from the point onto the line.
7. Draw a line 12 cm long. Make a point, Q, approximately 3 cm from one end. Draw a perpendicular at Q.
8.
 - (a) Draw a triangle ABC with side $AB = 8$ cm, $BC = 11$ cm and the angle at B, $\angle ABC = 35^\circ$.
 - (b) Bisect BC and call the midpoint M.
 - (c) Bisect AC and call the midpoint N.
 - (d) Bisect AB and call the midpoint P.
 - (e) Join AM, BN and CP. These are the **medians**.
 - (f) Mark the point of intersection of the medians, G. G is the **centroid** or **centre of gravity** of the triangle.
9.
 - (a) Draw triangle ABC as described in question 8.
 - (b) Bisect each angle of the triangle.
 - (c) Mark the point of intersection of these bisectors H.
 - (d) With centre H, draw a circle which just touches all three sides of the triangle. This circle is called the **incircle** and it is the **incentre**.
10.
 - (a) Draw triangle ABC as described in question 8.
 - (b) Perpendicularly bisect BC, AC and AB.
 - (c) Mark the point of intersection of these bisectors J.
 - (d) With centre J and radius JA, draw a circle. This circle should also pass through B and C. It is called the **circumcircle** and J is the **circumcentre**.
11.
 - (a) Draw triangle ABC as described in question 8.
 - (b) Drop a perpendicular from A onto BC, from B onto AC and from C onto AB. These perpendiculars are called **altitudes**.
 - (c) Mark the point of intersection of the altitudes K. K is called the **orthocentre** of the triangle.

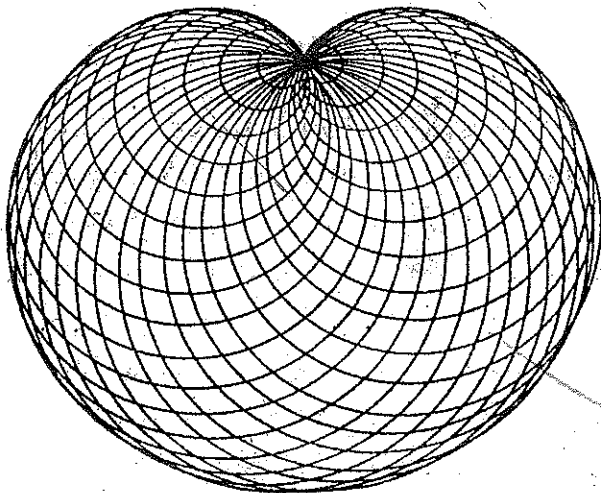
Exercise 3.6 Geometric Designs and Patterns

Here are some fun designs to draw using a pencil, ruler, compass and protractor. Each design has instructions to follow and a picture of the completed design.

Remember to colour the finished design since this will improve it. You can use the given pictures or invent your own colouring patterns.

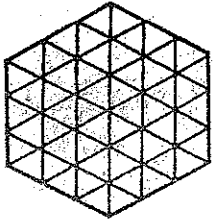
1. Cardioid

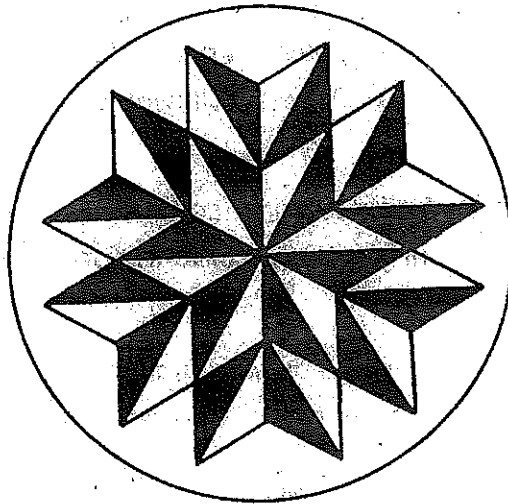
- (a) In the middle of your page draw a circle of radius 3 cm.
- (b) Using a protractor, carefully mark off points around the circle every 10° .
- (c) Label the lowest mark A.
- (d) Using each point marked on the circle as a centre, draw a new circle that passes through the point A.



2. 12 pointed Star

- (a) Draw a circle of radius 9 cm.
- (b) Using your compass (with radius 9 cm) mark 6 points evenly around the circle.
- (c) Join the 6 points, forming a hexagon.
- (d) Divide each line of the hexagon into 3 equal parts.
- (e) Draw the grid as shown.
- (f) Erase some lines to form the star.
- (g) Add extra lines to form the diamond shapes.
- (h) Colour your design.





3. Vanishing Circles

- (a) Draw a circle of radius 10 cm.
- (b) Draw lines from the centre to the circle at 20° intervals.
- (c) Using every second point of intersection on the circle, draw a semi-circle that just touches the two lines on either side.
- (d) From the original centre draw a circle that just touches each semi-circle.
- (e) Mark the points of intersection of the new circle and the 20° lines which lie between the semi-circles.
- (f) Using these points draw new semi-circles as before.
- (g) Repeat steps 4, 5 and 6 as often as possible.
- (h) Colour your design.

