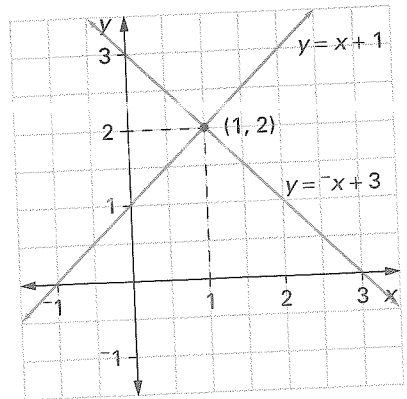


Solving simultaneous equations

The intersection of two linear equations can be shown on a graph as the intersection of two straight lines. At the point where the two straight lines meet, both linear equations have the same coordinates.

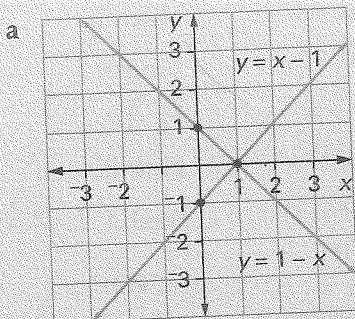
In this section you will use three methods to find the point of intersection of linear equations. To find the intersection of two linear equations using algebra is to solve the two equations 'simultaneously'.



Using graphs

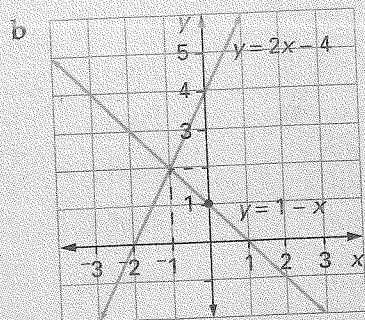
Examples

State the point of intersection of the following pairs of straight lines.



Solution

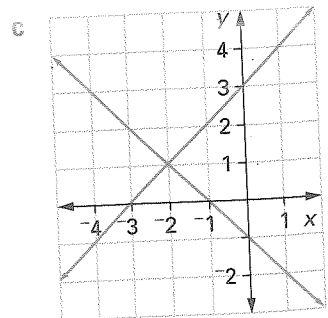
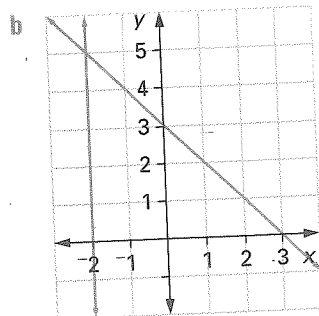
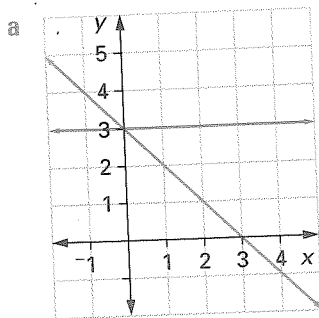
a The intersection point is $(1, 0)$.



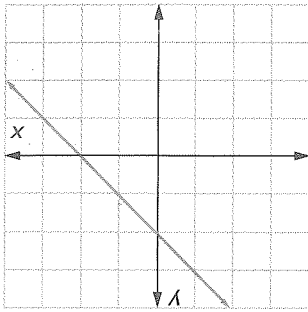
b The intersection point is $(-1, 2)$.

Exercise 5.11 Solving simultaneous equations 1

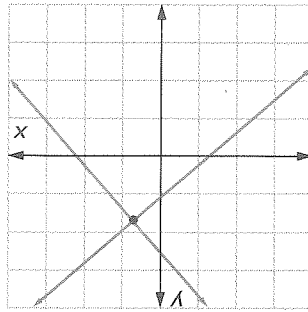
1 State the point of intersection for each of the following pairs of straight lines.



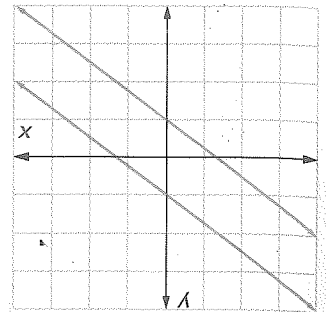
- 1 How many ways can three straight lines intersect? Sketch them.
- 2 How many ways can four straight lines intersect? Sketch them.
- 3 How many ways can five straight lines intersect? Sketch them.



Infinitely many points of intersection (parallel and touching)



One point of intersection (intersecting)



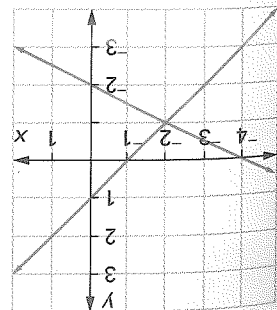
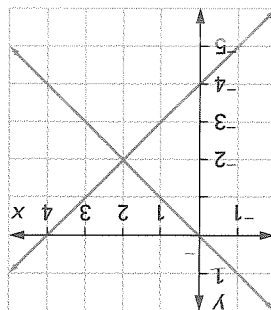
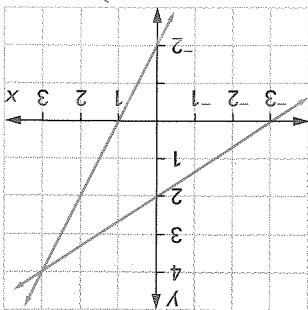
No points of intersection (parallel but not touching)

Two straight lines can intersect in three different ways.

Lines of intersection

PROBLEM SOLVING

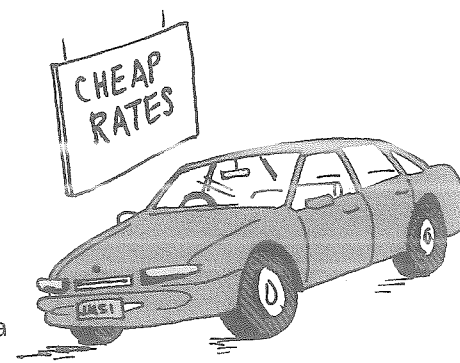
- 2 Find the point of intersection of each of the following pairs of straight lines by making an accurate graph.
 - a $y = 2$ and $y = 2x + 6$
 - c $y = -2$ and $x = 3$
 - e $y = -3x - 1$ and $x = 2$
- 3 Find the point of intersection of each pair of straight lines.
 - a $y = 2x - 4$ and $y = 3x + 6$
 - b $y = 2x + 1$ and $y = 3x - 2$
 - c $y = -x + 4$ and $y = 2x - 1$
 - d $y = -3x + 5$ and $y = -x + 3$
 - e $y = 3x + 6$ and $y = x - 4$
 - f $y = -3x + 4$ and $y = -5x - 2$



d

A cheaper rate is possible by hiring the car for \$20 less per day than the fixed rate, but paying 25 cents for each kilometre over 100 km per day.

- 1 How much does it cost to hire a Holden Commodore for 16 days at the fixed rate?
- 2 How much does it cost to hire a Mazda 323 for 7 days at the fixed rate?
- 3 How much does it cost to hire a Mazda 323 for 5 days at the cheaper rate?
- 4 Use a graph to find when it is best to hire a Mazda 323 using the fixed rate.
- 5 Use a graph to find when it is best to hire a Ford Fairmont Ghia using the fixed rate.



Using the method of substitution

To find the point of intersection using the method of substitution is to put the x or y terms of one equation into the other equation.

Example

Find the point of intersection between the straight lines $y = 4 - x$ and $2x + y = 5$ using the method of substitution.

Solution

Label each equation

$$y = 4 - x \quad [1]$$

$$2x + y = 5 \quad [2]$$

Substitute equation [1] into equation [2].

$$2x + (4 - x) = 5$$

$$2x + 4 - x = 5$$

$$x + 4 = 5$$

$$x = 1$$

expand brackets
add like terms

subtract 4 from both sides

Find the value for y by substituting $x = 1$ back into equation [1].

$$y = 4 - x$$

$$y = 4 - 1$$

$$y = 3$$

Hence, the point of intersection is $(1, 3)$.

Check the answer by substituting $(1, 3)$ back into both equations.

Exercise 5.12 Solving simultaneous equations 2

- 1 Solve the following pairs of linear equations by using the method of substitution.
 - a $y = x + 2$ and $3x + y = 6$
 - b $y = x - 3$ and $3x + y = 17$
 - c $y = x + 5$ and $5x - y = -1$
 - d $y = x - 3$ and $2x - y = 7$
 - e $y = x + 3$ and $2x + 3y = 19$
 - f $y = x - 1$ and $3x + 5y = 27$

To find the point of intersection using the method of elimination is to add or subtract the x - and y -values of each equation in a way which 'eliminates' all the x -terms or all the y -terms. It has the advantage of being quick and accurate, unlike drawing graphs.

Using the method of elimination

- 2 Solve the following pairs of linear equations by using the method of substitution.
- a $x = y + 1$ and $3x + 2y = 13$
 - b $x = 4 - y$ and $4x - 2y = 8$
 - c $x = 5 - y$ and $2x - 3y = 5$
 - d $x = 2y + 1$ and $3x + 5y = 25$
 - e $y = 3x + 4$ and $2x + 3y = 23$
 - f $y = 5 - 2x$ and $3x - 2y = 4$
 - g $y = 2x + 1$ and $2x + 3y = 11$
 - h $y = 5x + 3$ and $4x + 2y = 20$
 - i $y = 3x + 2$ and $4x - 5y = 1$
- 3 Solve the following pairs of linear equations by using the method of substitution.
- a $y = 2x - 1$ and $3x + 5y = -18$
 - b $x = y - 4$ and $2x - 3y = 8$
 - c $y = x + 5$ and $x - 4y = -2$
 - d $y = 5x - 1$ and $3x + 2y = 13$
 - e $y = 5x - 1$ and $3x + 2y = 13$
 - f $x = y + 2$ and $5x - 2y = 7$
 - g $x = y + 2$ and $5x - 2y = 7$
 - h $y = 2x + 3$ and $7x + 2y = 5$
 - i $y = 8x - 5$ and $5x - 6y = -13$

Examples

- 1 Find the point of intersection between the straight lines $y = 4 - x$ and $2x + y = 5$ using the method of elimination.

Solution

Label each equation $x - y = 4$ [1]

$$2x + y = 5$$

Adding equations [1] and [2] eliminates the value of y :

$$x - y = 4$$

[1]

$$+ 2x + y = 5$$

[2]

$$3x = 9$$

divide both sides by 3

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

Find the value for y by substituting $x = 1$ back into equation [1] or [2].

$$x - y = 4$$

[1]

$$3 - y = 4$$

subtract 3 from both sides

$$-y = 1$$

multiply both sides by -1

$$y = -1$$

Hence, the point of intersection is (3, -1).

Check the answer by substituting (3, -1) back into each equation.

Examples

- 2 Find the point of intersection between the straight lines $3x + 2y = 13$ and $4x - 3y = 6$ using the method of elimination.

Solution

Label each equation: $3x + 2y = 13$ [1]
 $4x - 3y = 6$ [2]

Multiply the equation [1] by 3 and equation [2] by 2 in order to eliminate y :

$$\begin{array}{r} 3x + 2y = 13 \quad \text{multiply by 3} \\ 4x - 3y = 6 \quad \text{multiply by 2} \\ \hline 9x + 6y = 39 \end{array}$$

$$\begin{array}{r} + 8x - 6y = 12 \\ \hline 17x = 51 \end{array}$$

divide both sides by 17

$$x = 3$$

To find the value of y , you substitute $x = 3$ into equation [1].

$$3 \times 3 + 2y = 13$$

$$9 + 2y = 13$$

$$2y = 4$$

$$y = 2$$

Hence the point of intersection is $(3, 2)$.

Check the answer by substituting $(3, 2)$ back into each equation.

Exercise 5.13 Solving simultaneous equations 3

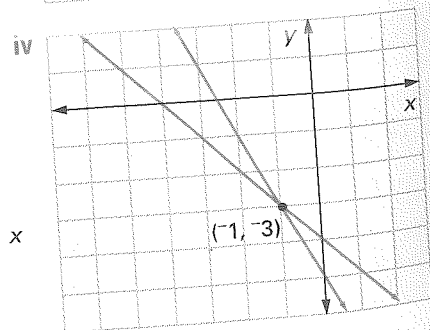
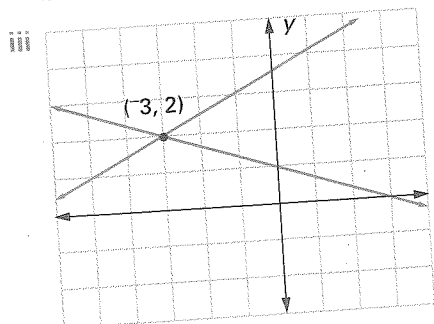
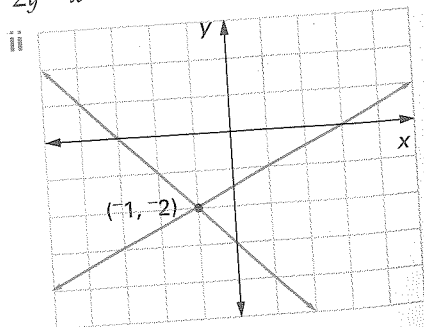
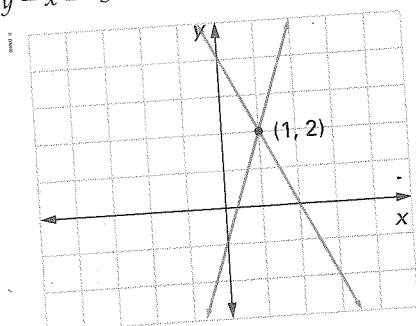
- 1 Match each point of intersection with the correct pair of equations.

a $y + x = -3$
 $2y - x = -3$

b $3x - y = 1$
 $2x + y = 4$

c $3y + x = 3$
 $2y - x = 7$

d $-x - y = 4$
 $2x + y = -5$



- 1 Bill and Finlay are brothers. The sum of their ages is 9. The difference between their ages is 3. Let Bill's age be represented by x and Finlay's age be represented by y . Write this information as two linear equations and then solve them simultaneously. Bill is older than Finlay.
- 2 A money box contains 50 coins, some are 20-cent pieces and the rest are 10-cent pieces. The total value of the coins is \$5.50. Set up two simultaneous equations and find the number of 20 cent and 10 cent coins in the money box.
- 3 Julie and Anne are gardeners. They buy two sorts of plants from a nursery, some for \$3 and the rest for \$5. If the total they spend is \$180, find the number of each plant purchased. The total number of plants is 50.
- 4 A test contains 42 questions, some are worth 2 marks and the rest are worth 3 marks. Find the number of each type of question, if the number of marks totals 100.
- 5 The basketball final is held in a stadium which can seat 15 000 people. All the tickets have sold, some for adults at \$25 and the rest for children at \$15. If the revenue from the tickets was \$315 000, find the number of adults and children who went to the game.
- 6 Find the price of each orange and apple from the following information: 9 oranges and 7 apples cost \$1.07, and 7 oranges and 9 apples cost \$1.01.

Word problems

PROBLEM SOLVING

- 2 Solve the following pairs of linear equations by using the method of elimination.
 - a $x + y = 6$ and $3x - y = 10$
 - b $4x + y = 23$ and $x - y = 2$
 - c $3x - y = 1$ and $2x + y = 4$
 - d $5x - y = 5$ and $x + y = 7$
 - e $y + x = -3$ and $2y - x = -3$
 - f $2y - x = 1$ and $3y - x = 0$
 - g $2x + 3y = 5$ and $5x - 3y = 2$
 - h $3x + 2y = 8$ and $x - 2y = 0$
 - i $-3x + 2y = -4$ and $5x - 2y = 8$
- 3 Solve the following pairs of linear equations by using the method of elimination.
 - a $x - y = 1$ and $4x + y = -10$
 - b $5x - 2y = 16$ and $-3x + 2y = -12$
 - c $4x + 3y = 18$ and $3x - y = 7$
 - d $3x + 2y = 12$ and $2x - y = 1$
 - e $3x - 5y = 14$ and $2x + y = 5$
 - f $5x + 2y = 30$ and $3x - y = 7$
 - g $4x - 5y = -21$ and $7x + y = 12$
 - h $2x - 4y = 2$ and $3x + y = -11$
 - i $5x + 3y = 1$ and $2x - 5y = -12$
- 4 Solve the following pairs of linear equations by using the method of elimination.
 - a $2x + 5y = -9$ and $3x - 4y = 21$
 - b $3x + 7y = -10$ and $5x - 2y = -3$
 - c $4x - 3y = 0$ and $3x + 4y = 25$
 - d $2x - 5y = -5$ and $3x + 3y = 24$
 - e $x - y = -4$ and $2x - 3y = 8$
 - f $3x - y = 2$ and $4x - 3y = 16$
 - g $5x - y = 1$ and $3x + 2y = 13$
 - h $2x + y = 5$ and $3x - 2y = 4$
 - i $x + y = 4$ and $x - y = 2$